Análisis Funcional, Teoría de Operadores y Ecuaciones de Evolución

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Spectral Properties of Rank one Perturbations of Unitary operators.

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Resumen

We analize spectral properties of some rank one perturbations of unitary operators with a cyclic vector. We also give conditions for unitary operators with a simple eigenvalue , in order to exhibit an almost exponential decay for the associated resonant state under perturbations. Main tools are the Aronszajn-Donoghue theory for rank one perturbations,and the reduction of the resolvent based on a unitary version of Feshbach-Livsic formula.

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Funciones zeta locales de Igusa y operadores pseudo-diferenciales sobre campos p-ádicos

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Resumen

En ésta charla se abordará una parte de la teoría de operadores pseudo-diferenciales p-ádicos relacionados con la función zeta local de Igusa. El objetivo es encontrar y estudiar las soluciones fundamentales de ecuaciones pseudo-diferenciales asociadas a estos operadores. Para ello, introduciremos la teoría preliminar en $\mathbb{Q}_p$ y, veremos además que, como consecuencia importante de un resultado de geometría algebraica, la función zeta local de Igusa admite una continuación analítica a todo el plano complejo. Esto último implica la existencia de soluciones fundamentales para operadores diferenciales p-ádicos con coeficientes constantes. Mostraremos también algunos operadores importantes y una técnica que se utilizará para hallar las soluciones fundamentales a las ecuaciones mencionadas.

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Difference Equations of type Volterra: A Qualitative Study

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Resumen

Let $\mathbf{X}$ be an arbitrary Banach space. We will study qualitative properties of the Volterra difference equation given by

$$u(n+1)=\lambda\sum_{j=-\infty}^{n}a(n-j)u(j)+g(n,u),\ n\in\mathbb{Z}, \tag{1}$$

where λ is a complex number, $a:\mathbb{N}\to\mathbb{C}$, and g are appropriate functions.

For this purpose we present a result of l^p-boundedness of the solution for linear Volterra difference equations, show a solution existence result l^p-boundedness of the Volterra functional difference equation given by

$$u(n+1)=\lambda\sum_{j=-\infty}^{n}a(n-j)u(j)+f(n,u_n),\ n\in\mathbb{Z}, \tag{2}$$

where $f:\mathbb{Z}\times\mathcal{B}\to\mathbf{X}$ is an appropriate function and $u_n:\mathbb{Z}_-\to\mathbf{X}$ is the pre-history function

We conclude by mentioning some qualitative properties of the set of solutions of the equation (1) as: Asymptotic behavior, continuity and compactness, also the adaptation of this theory to the Turelli-Hoffman-Schofield model for propagation of Wolbachia bacteria.

Joint work with:
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Linear Response Theory: An Analytic-Algebraic Approach

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Resumen

Linear response theory (LRT) is a tool with which one can study the response of systems that are driven out of equilibrium by external perturbations. In this talk I present a modern and systematic approach to LRT by combining analytic and algebraic ideas. The theory is robust and provides a tool to implement LRT for a wide array of systems like periodic and random systems in the discrete and the continuum. The mathematical framework of the theory is outlined firstly: the relevant von Neumann algebras, non-commutative L^p- and Sobolev spaces are introduced; the notion of isospectral perturbations and the associated dynamics and commutators are studied; their construction is then made explicit for various physical systems (quantum systems, classical waves). The final part is dedicated to a presentation of the proofs of the Kubo and Kubo-Streda formulas. The content of the talk is based on [1].

Joint work with:
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A class of Weierstrass-Enneper lifts of harmonic mappings

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Resumen

Bounds on the Schwarzian derivative of a holomorphic locally univalent function are known to imply the global univalence of the function and its quasiconformal extension to $\mathbb{C}$. This theory has been extended to harmonic mappings and their associated minimal surfaces in the works [1] and [2].

Assuming that the holomorphic part of a harmonic mapping has small Schwarzian derivative we find conditions on its dilatation which, in view of a criterion from [2], imply that the Weierstrass-Enneper lift of the mapping is univalent. A similar theorem is proved for harmonic mappings which are shears of holomorphic functions with small Schwarzian derivative and, in this case, tools from [3] allow us to deduce the univalence of the planar harmonic mapping, along with its quasiconformal extension to $\mathbb{C}$. We note that, more generally, the conformal map of any quasidisk admits shearings with sufficiently small dilatations.

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Systems of Schrodinger equations

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Resumen

We consider a time-dependent coupled system of Schrödinger equations,

$$i\,u_t = -\Delta u + V(x)u + g(x)v$$
$$i\,v_t = -\Delta v + W(x)v + g(x)u$$

both equations in the half line $x > 0$, with Dirichlet boundary condition at the origin. We assume that the potential V and W and the coupling term g are real valued bounded functions with compact support.

Let k_0 be a resonance for above system, that is $\Im k_0 < 0$, and $k_0^2 = \lambda - i\epsilon$, with λ and ϵ positive, are such that the corresponding stationary system has a non trivial outgoing solution, the so called resonant solution.

The resonant solution φ is increasing at ∞ so we truncate it to an interval $[0, R]$ containing the supports of the functions V, W and g. After normalization, we obtain a function ψ which should manifest the resonance phenomenon.

We prove that this is the case from the dynamical point of view, obtaining an approximate exponential decay for the expression,

$$\langle \psi, u(t) \rangle,$$

where $u(t)$ is the solution with initial value ψ. We follow a method due to Lavine, who studied resonances for a quantum particle in the half line

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Lebesgue regularity for nonlocal time-discrete equations with delays

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Resumen

In this talk, we provide a new and effective characterization for the existence and uniqueness of solutions for nonlocal time-discrete equations with delays, in the setting of vector-valued Lebesgue spaces of sequences.
This characterization is given solely in terms of the R-boundedness of the data of the problem, and in the context of the class of UMD Banach spaces.

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Resonancias en guías de ondas torcidas

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Resumen

En esta charla consideraremos el Laplaciano para una guía de ondas recta, la cual será torcida localmente. Se sabe que tal perturbación no crea nuevos valores propios. Sin embargo, es posible definir una extensión meromorfa de la resolvente del Laplaciano perturbado, la que nos permite mostrar que existe exactamente una resonancia cerca del ínfimo del espectro esencial. Calcularemos también el comportamiento asintótico de esta resonancia, en función del tamaño del torcimiento. Por último daremos una idea de como extender estos resultados para los "umbrales" superiores en el espectro del Laplaciano no perturbado

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Radial Solutions for a Class of Non-Local Equations

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Resumen

The scope of this talk is to show existence of radial solutions and regularity properties for a class of non local equations. We use the theory of Fourier multipliers for constructing suitable domains of the pseudo-differential operators naturally associated to the corresponding equations under consideration. On these domains the operators can be rigorously defined, and we prove existence of solutions belonging to these domains. We also include applications of the theory to equations of physical interest involving the fractional Laplace operator such as the Allen-Cahn equation.

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The generalised quantum Harmonic oscillator and its decoherence-free sub-algebra

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Resumen

We consider the definition of the generalised Quantum Harmonic Oscillator (QHO) introduced by Bath and Parthasarathy in [2]. It is well-known (see [1]) that all QMS suffers decoherence in its evolution. Loosely speaking, one says that "the quantum evolution becomes classical," this means that after some time, the dynamics concentrates on a commutative sub-algebra of observables. In [3] we characterized decoherence-free subalgebras where the evolution preserves its quantum structure. In general, these sub-algebras are trivial, nevertheless some physical systems do contain non trivial decoherence sub-algebras. More precisely, the decoherence-free subalgebra is the biggest (non commutative) where the semigroup act as a group of endomorphisms. The conference will show that the generalised QHO has a non trivial decoherence-free subalgebra.

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One-channel operators and a radial transfer matrix approach to spectral theory

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Resumen

We introduce the notion of a one-channel operator as a generalization of Jacobi operators and show how solutions to the formal eigenvalue equation and the spectral theory can be completely described by analyzing products of 2×2 transfer matrices. We use this technique to show existence of absolutely continuous spectrum for Anderson models on certain (partial) antitrees of more than 2-dimensional growth [1, 2]. Inspired from the specific form of these transfer matrices we introduce a general radial transfer matrix approach for finite-hopping Hermitian operators [3]. Instead of solving the eigenvalue equation the transfer matrix products serve as a linearization of a semi-group structure for Greens-function boundary data along 'spheres' starting from some root point. We hope that this technique can help to advance in some of the open problems of spectral theory for disordered systems.

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Asymptotic formulae for solutions of impulsive differential equations with piecewise constant argument of generalized type

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Resumen

In the late 70's, *A.D.Myshkis* noticed that there was no theory for differential equations with discontinuous argument of the form $x'(t) = f(t, x(t), x(h(t)))$; where, for example, $h(t) = [t]$, and he called these equations *Differential equations with deviating argument.* The systematic study of problems related to piecewise constant argument began early in the 80's with the works of *Cooke, Wiener* and *Shah.* They called these type of equations *Differential Equations with Piecewise Constant Argument* (in short *DEPCA*).

S. Busenberg and *K.L. Cooke* were the first to introduce a mathematical model that involved such types of deviated arguments in the study of models of vertically transmitted diseases, reducing their study to discrete equations. Since then, these equations have been deeply studied by many researchers of diverse fields like biomedicine, chemistry, biology, physics, population dynamics and mechanical engineering (see [4, 6]).

M. Akhmet considered the equation

$$x'(t) = f(t, x(t), x(\gamma(t))),$$

where $\gamma(t)$ is a *piecewise constant argument of generalized type*, that is, given $(t_k)_{k\in\mathbb{Z}}$ and $(\zeta_k)_{k\in\mathbb{Z}}$ such that $t_k < t_{k+1}, \forall k \in \mathbb{Z}$ with $\lim_{k\to\pm\infty} t_k = \pm\infty$, $t_k \leq \zeta_k \leq t_{k+1}$ and $\gamma(t) = \zeta_k$ if $t \in I_k = [t_k, t_{k+1})$. These equations are called *Differential Equations with Piecewise Constant Argument of Generalized Type* (in short *DEPCAG*). They have continuous solutions, even when $\gamma(t)$ is not. In the end of the constancy intervals they produce a recursive law, i.e, a discrete equation. That is the reason why these equations are called *hybrids*, because they combine discrete and continuous dynamics (see [9]).

In the DEPCAG case, when continuity at the endpoints of intervals of the form $I_k = [t_k, t_{k+1})$ is not considered, gives rise to *Impulsive Differential Equations with Piecewise Constant Argument of Generalized Type* (in short *IDEPCAG*)

$$\begin{aligned} x'(t) &= f(t, x(t), x(\gamma(t))), && t \neq t_k \\ \Delta x(t_k) &= Q_k(x(t_k^-)), && t = t_k \end{aligned} \tag{1}$$

where $x(\tau) = x_0$ (see [5, 11]).

This time we study the existence of an asymptotic equilibrium for (1). I.e, strongly based on certain integrability conditions, Gronwall-Bellman type inequality and the Banach's fixed point theorem, we prove that every solution of (1) with initial condition $x(a) = x_0$ where $a \geq \tau$ satisfies has the asymptotic formulae

$$x(t) = \xi + \mathcal{O}\left(\sum_{i=1}^{3}\int_t^\infty \lambda_i(s)ds + \sum_{t\leq t_k<\infty}\left(\mu_k^1 + \mu_k^2\right)\right). \tag{2}$$

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for some $\xi \in \mathbb{R}^n$, where λ and μ are Lipschitz constants related to f and Q_k respectively. Furthermore, if we consider the systems

$$\begin{aligned} y'(t) &= A(t)y(t) + g(t, y(t), y(\gamma(t))), \quad t \neq k \\ \Delta y(t_k) &= J_k y(t_k^-) + I_k(y(t_k^-)), \qquad t = k \in \mathbb{N} \end{aligned} \tag{3}$$

$$\begin{aligned} z'(t) &= A(t)z(t), \quad t \neq k \\ \Delta z(t_k) &= J_k z(t_k^-), \quad t = k \in \mathbb{N} \end{aligned} \tag{4}$$

then, under some conditions, any solution y of (3) satisfies the asymptotic formulae

$$x(t) = \Phi(t)\,[\nu + \epsilon(t)]\,, \nu \in \mathbb{R}^n, \quad \epsilon(t) \to 0 \quad \text{as } t \to \infty, \tag{5}$$

where Φ is the fundamental matrix of (4). I.e systems (3) and (4) are asymptotically equivalent and

$$y(t) = z(t) + \epsilon_0(t), \quad \epsilon_0(t) \to 0 \quad \text{as } t \to \infty.$$

Moreover, asymptotic equivalence includes the case of unbounded solutions.
The results are an extension of [7, 8] for the IDEPCAG case and, in particular, generalize the work done by Bereketoglu in [3] for a general piecewise constant argument. An example of a second order IDEPCAG will be shown.

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