DISTANCE DUAL IN GRAPH THEORY[1]

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Abstract: The intention of this work is to present a new concept of distance between graphs. We have been called this *dual distance* for the fact that every graph is realizable on the space $\mathbb{R}^3$. It is of observing that the idea of accomplishment of a graph on $\mathbb{R}^3$ presented here is completely original and it is the axis of our construction. Framed in the area of the Dynamics of Graphs, inside the Graph Theory, presently work will be used, preferably, simple and finite graph.

Key words: Simple and finite graph, Realizable Graph.

AMS subject classifications : 05C25 - 05C35.

1. Introduction : The graph to be considered will be in general simple and finite, graphs with a nonempty set of edges. For a graph G, V(G) denote the set of vertices and E(G) denote the set of edges. The cardinality of V(G) is called order of G and the cardinality of E(G) is called size of G. A (p,q) graph has order p and size q. Two vertices u and v are called neighbors if {u,v} is an edge of G. For any vertex v of G, denotes by N_v the set neighbors of v. To simplify the notation, an edge {x,y} is written as xy (or yx). Other concepts used in this work and not defined explicitly can be found in the references [l], [2], [3], [5], [9], [12].

2. Preliminary. Some essential concepts of this work are the following ones:

2.1. Realizable Graph [3],[[7], [8],[10]: One (p, q) grafo G is said realizable in $\mathbb{R}^3$ if it is possible to distinguish a collection of p different points of $\mathbb{R}^3$, that correspond to the vertices of G and a collection of q curves, disjoint two to two except possibly in the extreme points, that correspond to the edges of G such that if a curve *g* corresponds to the edge $e = uv$, then only the extreme points of *g* correspond to the vertices *u* and *v*.

In our work the concept will be in use of dualitation for that of realization, according to the following construction.

2.1.1.- Dual of a graph: For I will be denoted the class of the simple and finite graphs. If $h : V(\mathrm{G}) \to \mathbb{R}^3$ is a injective function, then the *dual* of G in $\mathbb{R}^3$, denoted by G* it is defined for

$\mathrm{G}^* = \{h(v) / v \in V(\mathrm{G})\} \bigcup (\bigcup_{uv \in E(\mathrm{G})} \mathrm{h(u)h(v)})$, where $h(u)h(v) = \{h(u) + t(h(v) - h(u)) / t \in [0,1]\}$.

This dualitation will have to satisfy the following condition: Each G□I admits one, and only one, realization G*, and $\forall G_1, G_2 \in \Omega : G_1{}^* = G_2{}^* \Leftrightarrow G_1 = G_2$

[1] Trabajo parcialmente financiado por la Dirección General de Investigación de la Universidad de Playa Ancha a través del Proyecto CNEI 010405 titulado " DIMENSIÓN DE LA ORBITA DE GRAFOS “

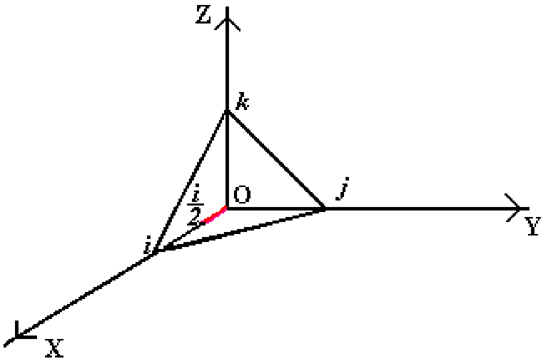
This identification defines the dual clase $\Omega^* = \{G^* \subset \mathsf{R}^3 / G \in \Omega\}$.

2.1.2.- Distance in I **[4],[6], [11]**: If $G_1, G_2 \in \Omega$, then d(G1,G2) = $\max_{u \in G_1^*, v \in G_2^*} d(u,v)$, is the distance between G1 and G2 . In this concept d is the usual distance in R^3.

Distance d will be called *dual.*

Examples:

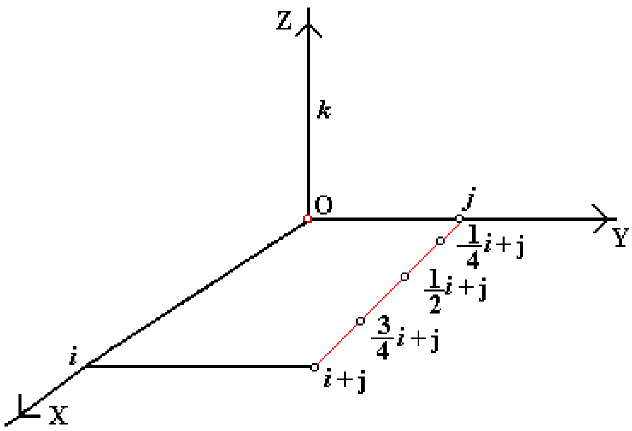
[1] If $G_1^* = \{i,j,k\} \cup ij \cup ik \cup jk$ and $G_2^* = \left\{0, \frac{i}{2}\right\} \cup 0\frac{i}{2}$, then $\mathrm{d}(G_1, G_2) = d(\frac{i}{2}, j) = \frac{\sqrt{5}}{2}$



[2]

If $G_1^* = \{0\}$ and $G_2^* = \left\{ j + mi / m \in \left\{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\right\}\right\} \cup \left(\bigcup_{t \in \{1,2,3,4\}} \left(j + \frac{(t-1)i}{4} \right)\left(j + \frac{ti}{4} \right) \right)$,

then $\mathrm{d}(G_1, G_2) = d(0, j+i) = \sqrt{2}$



3. Dual Distance . Now we will prove that the distance previously definite is exactly a distance which we will call *dual*.

3.1 Proposition: The function $d:\Omega\times\Omega\to$ □ such that d(G_1,G_2) = $\max\limits_{u\in G_1^*, v\in G_2^*} d(u,v)$, it is a distance on □.

Proof : $\forall G_1, G_2, G\in\Omega$:

(i) $\mathrm{d}(G_1,G_2)=0 \Leftrightarrow G_1=G_2$

Really:

$$\mathrm{d}(G_1,G_2)=0 \Leftrightarrow \max_{u\in G_1^*, v\in G_2^*} d(u,v)=0 \Leftrightarrow \forall u\in G_1^*, \forall v\in G_2^* : d(u,v)=0 \Leftrightarrow$$

$$\Leftrightarrow \forall u\in G_1^*, \forall v\in G_2^* : u=v \Leftrightarrow G_1^*=G_2^* \Leftrightarrow G_1=G_2$$

(ii) $\mathrm{d}(G_1,G_2)>0 \Leftrightarrow G_1\neq G_2$

In effect :

$$\mathrm{d}(G_1,G_2)=0 \Leftrightarrow \max_{u\in G_1^*, v\in G_2^*} d(u,v)>0 \Leftrightarrow \exists u\in G_1^*, \exists v\in G_2^* : d(u,v)>0 \Leftrightarrow$$

$$\Leftrightarrow \exists u\in G_1^*, \exists v\in G_2^* : u\neq v \Leftrightarrow G_1^*\neq G_2^* \Leftrightarrow G_1\neq G_2$$

(iii) $\mathrm{d}(G_1,G_2)=\mathrm{d}(G_2,G_1)$

The result is immediate for the uniqueness of the maximum.

(iv) $\mathrm{d}(G_1,G_2)\leq \mathrm{d}(G_1,G)+\mathrm{d}(G,G_2)$

Since $\mathrm{d}(G_1,G_2)=\max\limits_{u\in G_1^*, v\in G_2^*} d(u,v)$ and

$$d(u,v)\leq d(u,w)+d(w,v) \text{ then } \max_{u\in G_1^*, v\in G_2^*} d(u,v)\leq \max_{u\in G_1^*, w\in G^* v\in G_2^*}\big(d(u,w)+d(w,v)\big)\leq$$

$$\leq \max_{u\in G_1^*, w\in G^*} d(u,w)+\max_{w\in G^* v\in G_2^*} d(w,v)=d(G_1,G)+d(G,G_2)$$

Of (i) the (iv)th the asked is obtained.

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