LIMIT CYCLES AND INVARIANT PARABOLA IN A KUKLES POLYNOMIAL DIFFERENTIAL SYSTEM OF DEGREE THREE

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Abstract. In this paper we consider a class of Kukles polynomial differential system of degree three of the form

$$\dot{x} = -y, \quad \dot{y} = Q(x, y)$$

having an invariant parabola , where $Q(x, y)$ is a polynomial with real coefficients of degree 3. For these class of second-order differential equations, we show that for certain values of the parameters the invariant parabola coexists with a center. For other values it can coexist with one , two or three small amplitude limit cycles which are constructed by Hopf bifurcation. This result give an answer for the question given in [9].

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