

Weighted isoperimetric inequalities and applications to PDE

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Abstract

Let a measure μ be given by $d\mu = \varphi(x)dx$ where φ is a continuous positive function on \mathbb{R}^N . If M is a smooth set on \mathbb{R}^N then let $\mu(M)$ denote its μ -measure and $P_\mu(M)$ its μ -perimeter which is given by $\int_{\partial M} \varphi(x)dS$. We show the following isoperimetric inequality: If $\varphi(x) = e^{|x|^2/2}$, then among all smooth sets with fixed μ -measure, the ball centered at the origin gives the smallest μ -perimeter. We also obtain related inequalities for other weights, and we discuss Didos problem in a halfspace. We then use these results to obtain weighted Sobolev inequalities and a-priori estimates for solutions of elliptic PDE. This is joint work with A. Mercaldo, M.R. Posteraro and M.F. Betta (University of Napoli).

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