THE SCHWARTZ-CHRISTOFFEL MAPPING FOR POLYGONS WITH INFINITELY MANY SIDES

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It is as early as 1945 that Myrberg attempted to define a period matrix for an hyperelliptic curve of infinite genus and equation

$$y^2 = \pi_{i=1}^{\infty}(x - a_i) = g(x)$$

The road was pursued further by Heins (1950), Ahlfors and Sario (1960), Accola (1960), and Mc Kean and Trubowitz (1976).
In this last work the authors consider Korteweg-de Vries equation tied to the periodic spectrum of Hill's operator.

When we go back to basics however, with Myrberg, we find that much needs to be clarified in order to have a Riemann matrix in this setting, and to try to answer some of R. Rodrguez questions.
As it is easy to see, periods are related to integrals

$$f(z) = \int_a^z \frac{p(x)}{\sqrt{g(x)}} dx$$

and the conformal map therewith.

It is these maps, of Schwartz-Christoffel type, that we investigate here. Namely we show

i) Maps into infinite stairs or hairy half planes given for example by

$$\int \sqrt{tan\, x}\, dx \qquad , \qquad \int \frac{cos\, x}{\sqrt{sin\, x}}\, dx$$

ii) Maps into domains bounded by fractals, such as Sierpinsky snowflake, given by

$$\int \pi(1 - g^{4n})$$

or similar g expansions.

It is noteworthy that none of these maps, however natural, have been studied before, making it plausible to study most of the classical topics (such as Fuchsian groups) in this "infinite"setting.

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