Outline of the talk by Helmut Linde, PUC

Geometrically Induced Bound States in Waveguides
(based on: P. Exner, H. Linde, T. Weidl: "Lieb-Thirring inequalities for geometrically induced bound states", Lett. Math. Phys. 70 (2004), no. 1, 83-95)

Consider the Laplacian operator $-\Delta^\Omega$ on a straight tube $\Omega \subset \mathbb{R}^{n+1}$ with fairly general cross-section $\omega \subset \mathbb{R}^n$ and with Dirichlet boundary conditions on the edge. Its spectrum is $\sigma = \sigma_{ess} = [\lambda_0, \infty)$, where λ_0 ist the lowest eigenvalue of the Laplacian on ω. If we introduce local deformations of the tube or mixed boundary conditions, new eigenvalues of $-\Delta^\Omega$ may appear below λ_0. Over the last years, the asymptotic behavior of these eigenvalues in the limit of weakly bound modes has been studied thoroughly. Little is known though about the case of perturbations of finite strength. In our article we will show how operator-valued Lieb-Thirring inequalities can be used to tackle this problem. Our results can be applied to a variety of special cases, for example

- a straight strip in the plane with mixed boundary conditions,
- a straight strip with bulges,
- a circular tube with bulges.

For each of these we give estimates on the eigenvalues of the corresponding Laplacian operator. For the case of a strip or tube with bulges, our inequalities capture the correct asymptotic behavior in the limit of weak perturbations.