Small Functional Perturbations for Scalar Dynamic Equations in Time Scale

Samuel Castillo*
Departamento de Matemáticas. Facultad de Ciencias
Universidad del Bío-Bío. Casilla 5-C. Concepción - Chile
E-mail:scastill@ubiobio.cl

Manuel Pinto†
Departamento de Matemáticas. Facultad de Ciencias
Universidad de Chile. Casilla 653. Santiago - Chile
E-mail:pintoj@uchile.cl

Abstract

In this talk it will be considered scalar linear functional dynamic equations in Time Scale of the form

$$y^{\Delta}(t) = \lambda_0(t)y(t) + b(t, y),\ t \in \mathbb{T} \cap [0, +\infty[, \tag{1}$$

where $\mathbb{T}$ (the Time Scale) is a closed subset of $\mathbb{R}$ without upper bound,

$$y^{\Delta}(t) = \lim_{s \to t, s \in \mathbb{T} - \{\sigma(t)\}} \frac{y(\sigma(t)) - y(s)}{\sigma(t) - s},$$

$\sigma(t) = \inf\{\tilde{t} > t : \tilde{t} \in \mathbb{T}\}$, $\lambda_0 : \mathbb{T} \cap [0, +\infty[\to \mathbb{C}$ is a function such that $\int_{t_1}^{t_2} \tilde{\lambda}_0(s)ds$ exists for all $t_1, t_2 \geq 0$, $\tilde{\lambda}_0(t) = \lambda_0(t)$ for $t \in \mathbb{T}$, $\tilde{\lambda}_0(s) = \lambda_0(t)$ for $s \in [t, \sigma(t)[$ if $\sigma(t) > t$, $\{b(t, \cdot)\}_{t \geq 0}$ is a family of bounded linear functionals from the set of the essentially bounded functions $[-\tau, 0] \cup \mathbb{T} \to \mathbb{C}$ into $\mathbb{C}$ which satisfies smallness conditions to be given later. The goal is to extend the Haddock-Sacker Conjecture (see [9]) for the equation (1). That conjecture, which is a version of the Hartman-Wintner Asymptotic Theorem [10] for delayed differential equations, has motivated several works (see for example [1, 5, 6, 7, 8]). Results in dynamics equations in Time Scale unify differential and difference equations [2, 3, 4, 11]. Examples and calculations are given.

References

[1] O. Arino and I. Gyori, Asymptotic integration of delay differential systems, *J. Math. Anal. Appl.* 138 (1989), 311-327.

*Supported by Fundación Andes C-13760 and Fondecyt 1030535
†Supported by Fondecyt 103535

[2] B. Aulbach and S. Hilger, Linear dynamics processes with homogeneous time scales. *Nonlinear Dynamics and Quantum Dynamical Systems.* Akademie Verlag, Berlin, (1990), 9-20.

[3] M. Bohner and D. Lutz, Asymptotic behavior of dynamic equations on Time Scale. *J. of Diff. Eqs. and Appl.* 7(1) (2001), 21-50.

[4] M. Bohner and A. Peterson, *Dynamic Equations on Time Scales: an introduction with applications.* Birkhauser, Boston 2001.

[5] J. S. Cassell and Z. Hou, Asymptotically diagonal linear differential equations with retardation, *J. London Math. Soc.* (2) 47 (1993) 473-483.

[6] S. Castillo and M. Pinto, An asymptotic theory for nonlinear functional differential equations. *Comput. Math. Appl.* **44** (2002), no. 5-6, 763-775.

[7] S. Castillo and M. Pinto, Asymptotics of Scalar Functional Differential Equations, *Functional Differential Equations*, The Research Institute, The College of Judea and Samaria, Ariel Israel Vol **11**. n 1-2 (2004), 29-36.

[8] I. Györi and M. Pituk, L^2-Perturbation of a linear delay differential equation, *J. Math. Anal. Appl.,* **195** (1995), 415-427.

[9] J.R. Haddock and R.J. Sacker, Stability and asymptotic integration for certain linear systems of functional differential equations, *J. Math. Anal. Appl.* 76 (1980), 328-338.

[10] P. Hartman and A. Wintner, Asymptotic integration of linear differential equations, *Amer. J. Math.* 77 (1955), 48-86.

[11] S. Hilger, Analysis on measure chains- an unified approach to continuous and discrete calculus. *Results Math., 18* (1990), 18-56.