2 Análisis Numérico

1. **Expositor:** Dalidet Sanhueza
   afiliación: Universidad del Bío-Bío
   Título: An exactly divergence-free finite element method for a bio-convection flows model
   Resumen: In this talk we present and analyze a finite element scheme yielding divergence-free velocities for a stationary bioconvective flows problem. The model consists of a Navier-Stokes type-system coupled to a cell conservation equation describing the motion of micro-organisms in an incompressible, viscous, culture fluid. A priori estimates for weak solutions to the corresponding variational formulation are derived and employed to state existence results by mean of a fixed–point approach and the Shauder Theorem. The proposed numerical scheme is based, on the one hand, on the standard interior penalty technique and an upwind approach for the nonlinear convective term for the fluid equations and, on the other hand, on a Galerkin discretization for the concentration equation. The finite element spaces that constitute the discrete scheme are given by the divergence-conforming Brezzi-Douglas-Marini (BDM) elements of order k for the velocity, discontinuous elements of order $k-1$ for the pressure and Lagrange elements of order k for the concentration. Existence and uniqueness results are shown and stated rigorously as well as optimal order a priori error estimates. We present a numerical example that backs up the theoretical expected convergence rates as well as the performance of the proposed technique. Joint work with:
   Eligio Colmenares[1], Departamento de Ciencias Básicas, Universidad del Bío-Bío, Chillán, Chile.
   Matias Garrido[2], Departamento de Matemática, Universidad del Bío-Bío, Concepción, Chile.

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2. **Expositor:** Eligio Colmenares
afiliación: Universidad del Bío-Bío
Título:A mass-conservative finite element method for a thermo- bio-convective flows model
Resumen: In this work, a finite element method for a thermo-bio-convective flow of gravitactic microorganisms in a porous media is introduced and analyzed. This method holds the divergence-free property in the discrete formulation. The model consists of a system of partial differential equations that relates the velocity and pressure of a Darcy-Brinkman fluid coupled with an advection-diffusion equation and a cellular conservation equation describing temperature and the concentration of the micro-organisms, respectively. Both aspects, theoretical and computational, are considered. The proof of stability, consistency, and convergence is boarded on a theoretical study of the numerical method, following energy estimates and the finite element theory. A few of numerical experiments allows to verify the theoretical results of the a priori error estimations and the convergence order with respect to the discretization parameter.

Joint work with:
Wilfredo Angulo[3], Departmento de Matemática, Universidad San Francisco de Quito, Quito, Ecuador.
Marco Estrada[4], Departmento de Matemática, Escuela Superior Politécnica del Litoral, Guayaquil, Ecuador.
Dany De Cecchis De Leon[5], Departmento de Matemática, Escuela Superior Politécnica del Litoral, Guayaquil, Ecuador.

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3. **Expositor:** Miguel Serón
   afiliación: Universidad del Bío-Bío
   Título:A conforming finite element method for a nonisothermal fluid-membrane interaction
   Resumen: In this paper, we propose and analyze a conforming finite element method for a mathematical model present, for instance, in desalination processes based on membrane distillation, consisting of the coupling of fluid flow with porous media flow subject to a heat source. The flows are governed by a natural convection model in the free fluid region, also known as Boussinesq model, and a Darcy-Heat coupled system in the porous media region. The coupled system is complemented by suitable boundary conditions on the exterior boundary and a set of transmission conditions on the interface given by mass conservation, balance of normal forces, the Beavers–Joseph–Saffman law, and the continuity of the heat flux and the fluid temperature. We consider the standard velocity-pressure-temperature variational formulation for the Boussinesq system, and a dual-mixed scheme coupled with a primal formulation for the Darcy and Heat equations, respectively, in the porous medium region. The latter yields the introduction of the trace of the porous medium pressure as a suitable Lagrange multiplier. As for the associated Galerkin scheme we employ Bernardi–Raugel and Raviart–Thomas elements for the velocities, piecewise constant elements for the pressures, continuous piecewise linear functions for the temperatures and continuous piecewise linear functions for the aforementioned the Lagrange multiplier. We prove well-posedness for both, the continuous and discrete schemes and derive the corresponding error estimates. Finally, we report some numerical examples confirming the predicted rates of convergence, and illustrating the performance of the method.

   Joint work with:
   Jessika Camaño[6], Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Concepción, Chile.
   Ricardo Oyarzúa[7], GIMNAP-Departamento de Matemática, Universidad del Bío-Bío.
   Manuel Solano[8], Departamento de Ingeniería Matemática, Universidad de Concepción, Concepción, Chile.

4. **Expositor:** Lady Angelo
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   Título:A five-field mixed formulation for stationary magnetohy- drody-

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namic flows in porous media
Resumen: noindent We introduce and analyze a new mixed variational formulation for a stationary magnetohydrodynamic flows in porous media problem, whose governing equations are given by the steady Brinkman–Forchheimer equations coupled with the Maxwell equations. Besides the velocity, magnetic field and a Lagrange multiplier asssociated to the divergence-free condition of the magnetic field, a convenient translation of the velocity gradient and the pseudostress tensor are introduced as further unknowns. As a consequence, we obtain a five-field Banach spaces-based mixed variational formulation, where the aforementioned variables are the main unknowns of the system. The resulting mixed scheme is then written equivalently as a fixed-point equation, so that the well-known Banach theorem, combined with classical results on nonlinear monotone operators and a sufficiently small data assumption, are applied to prove the unique solvability of the continuous and discrete systems. In particular, the analysis of the discrete scheme requires a quasi-uniformity assumption on mesh. The finite element discretization involves Raviart–Thomas elements of order $k \geq 0$ for the pseudostress tensor, discontinuous piecewise polynomial elements of degree k for the velocity and the translation of the velocity gradient, Nédélec elements of degree k for the magnetic field and Lagrange elements of degree $k+1$ for the associated Lagrange multiplier. Stability, convergence, and optimal *a priori* error estimates for the associated Galerkin scheme are obtained. Numerical tests illustrate the theoretical results.

Joint work with:
Jessika Camaño[9], Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Concepción, Chile and CI^{2}MA, Universidad de Concepción, Concepción, Chile.
Sergio Caucao[10], Departamento de Matemática y Física Aplicadas, Universidad Católica de la Santísima Concepción, Concepción, Chile.

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[9]Partially supported by ANID-Chile through the project Centro de Modelamiento Matemático (FB210005) and Fondecyt project 1180859, e-mail: jecamano@ucsc.cl
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5. **Expositor:** Johann Esparza
afiliación: Universidad de Concepción
Título: An augmented mixed FEM for the convective Brinkman– Forchheimer problem: a priori and a posteriori error analysis
Resumen: We propose and analyze an augmented mixed finite element method for the pseudostress-velocity formulation of the stationary convective Brinkman–Forchheimer problem in R^d, $d \in \{2, 3\}$. Since the convective and Forchheimer terms forces the velocity to live in a smaller space than usual, we augment the variational formulation with suitable Galerkin type terms. The resulting augmented scheme is written equivalently as a fixed point equation, so that the well-known Schauder and Banach theorems, combined with the Lax–Milgram theorem, allow to prove the unique solvability of the continuous problem. The finite element discretization involves Raviart–Thomas spaces of order $k \geq 0$ for the pseudostress tensor and continuous piecewise polynomials of degree $\leq k + 1$ for the velocity. Stability, convergence, and *a priori error* estimates for the associated Galerkin scheme are obtained. In addition, we derive two reliable and efficient residual-based *a posteriori* error estimators for this problem on arbitrary polygonal and polyhedral regions. The reliability of the proposed estimators draws mainly upon the uniform ellipticity of the form involved, a suitable assumption on the data, a stable Helmholtz decomposition, and the local approximation properties of the Clément and Raviart–Thomas operators. In turn, inverse inequalities, the localization technique based on bubble functions, and known results from previous works, are the main tools yielding the efficiency estimate. Finally, some numerical examples illustrating the performance of the mixed finite element method, confirming the theoretical rate of convergence and the properties of the estimators, and showing the behaviour of the associated adaptive algorithms, are reported. In particular, the case of flow through a 2D porous media with fracture networks is considered.

Joint work with:
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6. **Expositor:** Sergio Caucao
afiliación: Universidad Católica de la Santísima Concepción
Título: A mixed FEM for the coupled Brinkman–Forchheimer/Darcy problem
Resumen: This work develops the *a priori* analysis of a mixed finite element method for the filtration of an incompressible fluid through a non-deformable saturated porous medium with heterogeneous permeability. Flows are governed by the Brinkman–Forchheimer and Darcy equations in the more and less permeable regions, respectively, and the corresponding transmission conditions are given by mass conservation and continuity of momentum. We consider the standard mixed formulation in the Brinkman–Forchheimer domain and the dual-mixed one in the Darcy region, and we impose the continuity of the normal velocities by introducing suitable Lagrange multiplier. The finite element discretization involves Bernardi–Raugel and Raviart–Thomas elements for the velocities, piecewise constants for the pressures, and continuous piecewise linear elements for the Lagrange multiplier. Stability, convergence, and *a priori* error estimates for the associated Galerkin scheme are obtained. Numerical tests illustrate the theoretical results.

Joint work with:
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7. **Expositor:** Francisco Fuica
afiliación: Pontificia Universidad Católica de Chile
Título:A posteriori error estimates for a distributed optimal control problem of the stationary Navier–Stokes equations
Resumen: In two and three dimensional Lipschitz, but not necessarily convex, polytopal domains, we propose and analyze a posteriori error estimators for an optimal control problem involving the stationary Navier–Stokes equations; control constraints are also considered. We devise two strategies of discretization: a semidiscrete scheme where the control variable is not discretized and a fully discrete scheme where the control is discretized. For each solution technique, we design an a posteriori error estimator that can be decomposed as the sum of contributions related to the discretization of the state and adjoint equations and, additionally, the discretization of the control variable for when the fully discrete scheme is considered. We prove that the devised error estimators are reliable and also explore local efficiency estimates. Numerical experiments reveal a competitive performance of adaptive loops based on the devised a posteriori error estimators.

Joint work with:
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8. **Expositor:** Paulina Sepúlveda
afiliación: Pontificia Universidad Católica de Valparaíso
Título:A Deep $r-$Adaptive First-Order System Least Squares method for elliptic PDEs
Resumen: In recent years, there has been a rise in the use of Deep Learning (DL) to solve Partial Differential Equations (PDEs). In this work, we aim to improve numerical solutions of first-order formulations of elliptic PDEs using a DL mesh adaptive strategy. In particular, we combine the advantages of the $r-$Adaptive DL mesh-based method presented in [1] with the Deep First-Order Systems Least Square (FOSLS) method proposed in [2]. Some features of the FOSLS method are: symmetrizes and stabilizes the original problem, obtains the approximation of the flux without post-processing, and allows solutions with discontinuous gradients. An advantage of the $r-$Adaptive DL method is that it simultaneously optimizes a neural network (NN) with the mesh-node locations and the piece-wise polynomial approximated solution. Numerical advantages of the $r-$ adaptive method for first-order systems compared to solutions computed on a uniform mesh will be presented for diffusion-advection and diffusion-reaction test problems.

Joint work with:
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9. **Expositor:** Thomas Führer
afiliación: Pontificia Universidad Católica de Chile
Título: Some remarks on mixed FEM: Quasi-optimality, singular data, postprocessing
Resumen: In this presentation we conisder a lowest-order mixed FEM for the Poisson problem. The classic formulation of mixed FEM does not allow loads that are not square-integrable. We propose to replace the load by a regularized load defined by an appropriate projection operator onto elementwise constants in H^{-1}, see [1]. The resulting mixed FEM can deal with singular data and we prove that it is quasi-optimal with respect to a weaker norm. Furthermore, we show that the method is also quasi-optimal in the flux with respect to L^2 norms. Such a result for the standard mixed FEM is only valid with additional restrictions. We provide a construction for a projection operator that is based on the dual of a novel weighted Clément quasi-interpolator. The latter locally recovers a continuous elementwise affine approximation from an elementwise constant approximation. Local postprocessing techniques for the primal variable are an essential tool in mixed FEM but for the lowest-order case require regular data (more than L^2 regularity) to prove optimal rates, see [2]. Here, we show for the regularized mixed FEM that such a regularity assumption can be removed yielding optimal rates for the postprocessed solution. Various numerical experiments are presented which support our theoretical findings.

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10. **Expositor:** Manuel A. Sánchez
afiliación: Pontificia Universidad Católica de Chile
Título: Combining finite element space-discretizations with symplectic time-marching schemes for linear Hamiltonian systems

Resumen: We provide a short introduction to the devising of a special type of methods for numerically approximating the solution of Hamiltonian partial differential equations. These methods use Galerkin space-discretizations which result in a system of ODEs displaying a discrete version of the Hamiltonian structure of the original system. The resulting system of ODEs is then discretized by a symplectic time-marching method. This combination results in high-order accurate, fully discrete methods which can preserve the invariants of the Hamiltonian defining the ODE system. We restrict our attention to linear Hamiltonian systems, as the main results can be obtained easily and directly, and are applicable to many Hamiltonian systems of practical interest including acoustics, elastodynamics, and electromagnetism. After a brief description of the Hamiltonian systems of our interest, we provide an introduction to symplectic time-marching methods for linear systems of ODEs which does not require any background on the subject. Finally, we consider the case of finite-element space discretizations. The emphasis is placed on the conservation properties of the fully discrete schemes. We end by describing ongoing work.

Joint work with:
Bernardo Cockburn[17], School of Mathematics, University of Minnesota, Minneapolis, MN 55455, USA
Shukai Du[18], Department of Mathematics, University of Wisconsin-Madison, WI 53706, USA.

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11. **Expositor:** Franz Chouly
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Título: Finite element approximations for the elastoplastic torsion problem

Resumen: We study the elastoplastic torsion problem, in dimension $n \geq 1$, and in a polytopal, convex or not, domain. In the physically relevant case where the source term is a constant, this problem can be reformulated using the distance function to the boundary. We present two discretizations based upon the aforementioned reformulation. The first one is a discrete variational inequality where the constraint is enforced at the mesh nodes. The second one is a Nitsche method. This second method has two advantages: 1) it leads to optimal error bounds in the natural norm, for Lagrange finite elements of order one or two, and even for nonconvex domains; 2) it is easy to implement within most of finite element libraries.

Joint work with:

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Patrick Hild[20], Institut de Mathématiques de Toulouse, UMR CNRS 5219, Université Paul Sabatier, 118 route de Narbonne, 31062 Toulouse Cedex 9, France.

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12. **Expositor:** Mauricio Sepúlveda Cortés

afiliación: Universidad de Concepción

Título: Inverse Problem for an intestinal crypt model

Resumen: We consider an intestinal crypt model including microbiota-derived regulations [4]. The simplified model considers a coupled system of 2 degenerate parabolic equations with cross diffusion whose unknowns are the density of progenitor cells (PC) and stem cells (SC). Additionally, the density of deep crypt secretory (DCS) cells acts as a function that we can assume to be known and that is known to affect the population dynamics in the crypt. The inverse problem consists in determining the parameters that define the shape of the density function of the DCS cells

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(slopes and position), from partial measurements of stem and progenitor cells. For this, we propose a classical method of adjoint state [1, 2, 3].

The general intestinal crypt model (considering 4 cell types) was introduced by Beatrice Laroche from INRAE, France, and her PhD student Marie Haghebaert who has used BGK schemes to successfully simulate the dynamics of the phenomenon. [4]

Joint work with:
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Marie Haghebaert [21] Université Paris-Saclay, INRAE, MaIAGE, 78350 Jouy-en-Josas, France.

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13. **Expositor:** Ruben Caraballo Diaz
afiliación: Universidad del Bío-Bío
Título: Augmented finite element method for Navier-Stokes equations with variable viscosity
Resumen: This talk deals with the analysis of an augmented mixed finite element method in terms of velocity-vorticity-pressure for the Navier-Stokes equations. The weak formulation is based on the introduction of suitable least squares terms arising from the constitutive equation relating the aforementioned unknowns and from the incompressibility condition. We show the well posedness of the continuous and discrete formulations.

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In addition, a priori error estimates and the corresponding rates of convergence are given. Finally, we report a set of numerical examples illustrating the behaviour of the proposed scheme.

Joint work with:
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14. **Expositor:** Pablo Venegas
afiliación: Universidad del Bío-Bío
Título: Some remarks on the numerical approximation of axisym- metric problems
Resumen: The aim of this talk is to study the numerical approximation of two axisymmetric problems related to acoustic and eddy current models. We show that the classical approach used in the cartesian setting is not the most appropriate to approximate these problems when the finite element method is used to discretize the corresponding weak formulations. We propose alternative weak formulations of the problems which allows us to avoid this drawback. Discretizations based on the same finite elements

are proposed and analyzed. Quasi-optimal order of convergence is proved. Finally, we report numerical results which allow us to confirm the theoretical estimates and to assess the performance of the proposed methods.

Joint work with:

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15. **Expositor:** Alberth Silgado
afiliación: Universidad del Bío-Bío
Título: A virtual element scheme for the unsteady Boussinesq equations
Resumen: In this talk, we develop a fully-discrete virtual element approximation for the nonstationary Boussinesq system in stream-function–temperature form. The spatial variables are discretized by using the C^1- and C^0-conforming virtual element approaches, while a backward Euler scheme is used for the temporal variable. We provide the well-posedness and unconditional stability of the fully-discrete scheme. Moreover, we derive error estimates in $L^2(H^2)$- and $L^2(H^1)$-norms for the stream-function and temperature fields, respectively. Finally, we present some numerical experiments.

Joint work with:

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16. **Expositor:** Patrick Vega
afiliación: Pontificia Universidad Católica de Valparaíso
Título: Frequency–explicit a posteriori error estimates for discon- tinuous Galerkin discretizations of Maxwell's equations
Resumen: We propose a new residual-based a posteriori error estimator for discontinuous Galerkin discretizations of time-harmonic Maxwell's equations in first-order form. We establish that the estimator is reliable and efficient, and the dependency of the reliability and efficiency constants on the frequency is analyzed and discussed. The proposed estimates generalize similar results previously obtained for the Helmholtz equation and conforming finite element discretization of Maxwell's equations. In addition, for the discontinuous Galerkin scheme considered here, we also show that the proposed estimator is asymptotically constant-free for smooth solutions. We also present two-dimensional numerical examples that highlight our key theoretical findings and suggest that the proposed estimator

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is suited to drive h- and hp-adaptive iterative refinements.

Joint work with:
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17. **Expositor:** Abner H. Poza
afiliación: Universidad Católica de la Santísima Concepción
Título: A stabilized finite element method for the Stokes–Temperature coupled problem
Resumen: In this talk, we introduce and analyze a new stabilized finite element scheme for the Stokes–Temperature coupled problem. This new scheme allows equal order of interpolation to approximate the quantities of interest, i.e. velocity, pressure, temperature, and stress. We analyze an equivalent variational formulation of the coupled problem inspired by the ideas proposed in [2]. The existence of the discrete solution is proved, decoupling the proposed stabilized scheme and using the help of continuous dependence results and Brouwer's theorem under the standard assumption of sufficiently small data. Optimal convergence is proved under classic regularity assumptions of the solution. Finally, we present some numerical examples to show the quality of our scheme, in particular, we compare our results with those coming from a standard reference in geosciences described in [5].

Joint work with:
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18. **Expositor:** Manuel Solano
afiliación: Universidad de Concepción
Título:A priori and a posteriori error analysis of an unfitted HDG method for semi-linear elliptic problems
Resumen: We present *a priori* and *a posteriori* error analysis of a high order hybridizable discontinuous Galerkin (HDG) method applied to a semi-linear elliptic problem posed on a piecewise curved, non polygonal domain. This problem comes from an application to plasma physics, where the magnetic equilibrium in axisymmetric fusion reactors can be described in terms of the solution of an equation of this type.

We approximate Ω by a polygonal subdomain Ω_h and propose an HDG discretization, which is shown to be optimal under mild assumptions related to the non-linear source term and the distance between the boundaries of the polygonal subdomain Ω_h and the true domain Ω. Moreover, a local

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non-linear post-processing of the scalar unknown is proposed and shown to provide an additional order of convergence. A reliable and locally efficient *a posteriori* error estimator that takes into account the error in the approximation of the boundary data of Ω_h is also provided.

Joint work with:
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19. **Expositor:** Esteban Henríquez
afiliación: Universidad de Concepción
Título: An unfitted HDG method for a distributed optimal control problem
Resumen: We analyze a high order hybridizable discontinuous Galerkin (HDG) method for an optimal control problem where the computational mesh does not necessarily fit the domain. The method is based on transferring the boundary data to the computational boundary by integrating the approximation of the gradient. We prove optimal order of convergence in the L^2-norm for all the variables of the state and adjoint problems, and the control variable as well. More precisely, order h^{k+1} if the local discrete spaces are constructed using polynomials of degree at most k on a triangulation of meshsize h. We present numerical experiments illustrating the

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performance of method.

Joint work with:
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20. **Expositor:** Isaac Bermúdez Montiel
afiliación: Universidad de Concepción
Título: An HDG method for Stokes/Darcy coupling in dissimilar meshes
Resumen: In many different applications, interfaces divide the domain of interest $\Omega \subset \mathbb{R}^{\mathrm{d}}$, where $\mathrm{d} = 2, 3.$, into several subdomains on which the governing equations and/or boundary conditions are diffe-rent. As the geometrical complexity and required spatial sampling of the subdomains may vary significantly, it is not uncommon to mesh the subdomains separately using different mesh sizes. In this work, we present and analyze a Hybridizable Discontinuous Galerkin (HDG) method for the problem posed by the coupling of the Stokes and Darcy equations, whose domains are discretized by two independent subdomains with different meshes. This causes non-conformity at the intersection of the subdomains or leaves a gap (unmeshed region) between them. To properly couple the two different discretizations, the proposed transmission conditions are based on mass conservation and equilibrium normal forces for matching meshes. For non-matching meshes, we use the transfer technique of the numerical trace/flux and extrapolate the approximate flux in both meshes. Furthermore, we establish the well-posedness of the method and error estimates to show the stability of the HDG method. Finally, we demonstrate the capacities of the method by presenting numerical experiments that validate our theory.

Joint work with:

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