3 Biomatemática

1. **Expositor:** Gerard Olivar-Tost
   Afiliación: Departamento de Ciencias Naturales y Tecnología
   Universidad de Aysén
   Coyhaique, Chile
   Título: Biological Control of the Coffee Berry Borer Hypothenemus hampei through Ant Predation
   Resumen: Coffee is a relevant agricultural product in global economy, with the amount and quality of the bean being seriously affected by the Coffee Berry Borer (Hypothenemus hampei (Ferrari), CBB)[1], its principal plague. One of the ways to counter this beetle is through biological controllers, like ants (Hymenoptera: Formicidae), some of which are characterized by naturally inhabiting coffee plantations and feeding on CBB in all their life stages. Considering the foregoing, the study describes a predator-prey interaction between these two insects, through a mathematical model based on ordinary differential equations, where state variables correspond to adult CBBs, immature CBBs, and ants from one species without specifying that they have among their feeding habits that of preying on the CBB, both in adult stages as in immature stages. The system's equilibrium points were determined and its stability was studied through qualitative theory and bifurcation theory and numerical simulations were performed to illustrate the stability results. The work permits determining species coexistence conditions, as well as conditions to eradicate the plague through the biocontrol action in combination with other actions focused on eliminating only adult CBBs.

   ## References

   [1] Johnson, M. A.; Ruiz-Diaz, C. P.; Manoukis, N. C.; Verle Rodrigues, J. C., *Coffee berry borer (Hypothenemus hampei), a 468 global pest of coffee: Perspectives from historical and recent invasions, and future priorities*, Insects **12**, (2020). 882.

2. **Expositor:** Stefan Berres
   Afiliación: Departamento de Sistemas de Información
   Universidad del Bío-Bío
   Concepción, Chile
   Título: Mathematical modeling of herbal extract release from biopolymer membranes
   Resumen: The controlled release of a herbal extract from biopolymer membranes can be used for the treatment of skin lesions. In the experimental study, release profiles where obtained from four different types of membranes that are classified as non-porous and porous membranes.

In a first modelling approach, the experimental data of release kinetics were fitted to five classical models from the literature, that correspond to linear functions, power functions (in the literature known as: Higuchi and Korsmeyer-Peppas model) and exponential functions (here: Weibull function). The best models are identified by optimizing a cost function that compares different models to the experimental data [1].

Yet, the modelling challenge consists of finding a better description of the diffusion within a swelling membrane and through a water filled network mesh, eventually considering that non-porous membranes have a fractal geometry and that porous membranes have highly disorganized structures. Therefore, in a second modelling approach, the model is formulated in terms of a diffusion equation, where the diffusion coefficient depends on the extract concentration. In the presentation, different modelling approaches and simulation results are discussed.

References

[1] Concha, L.; Resende Pires, A.L.; Moraes, A.M.; Mas-Hernández, E.; Berres, S.; Hernández-Montelongo, J., *Cost Function Analysis Applied to Different Kinetic Release Models of Arrabidaea chica Verlot Extract from Chitosan/Alginate Membranes*, Polymers **14**, (2022). 1109. https://doi.org/10.3390/polym14061109

[2] Concha, Luis, *Simulación de la liberación de extracto de Arrabidaea chica desde membranas hinchables de quitosana/alginato*, Actividad de Formación Equivalente (AFE); Universidad Católica de Temuco, (2021). http://tiny.cc/afe-liberacion

3. **Expositor:** Viviana Rivera-Estay
Afiliación: Doctorado en Modelamiento Matemático Aplicado
Universidad Católica del Maule
Talca, Chile
Título: Interactions between factors determining a biological invasion: mathematical modelling of a predator-prey system
Resumen: Biological invasions threaten biodiversity as they are considered one of the main causes of species extinction [1, 2]. Determining conditions under which an introduced exotic species achieves invasion success will provide a broader understanding of biological invasions and thus raise theoretical projections supporting future research [3]. The potential of phenotypic change has a relevant role in the success of an invasion, [4, 5] in fact, it could help native species to mitigate the negative impacts of an invasion [6, 7], and thus reduce the invasiveness of the site. However, it could give an advantage to establishment of an invasive species, that is, increase the invasiveness of the exotic species [8]. On the other hand, the success of an invasion depends on the introduction events and the number

of individuals introduced in each of these events, that is the propagule pressure [9, 10, 11]. Since exotic predators are the most disturbing introduced species [12, 13], this project aims to determine conditions under which an exotic predator achieves invasion success in a predator-prey system. It will be possible to analyze the interplay between determinant factors of a biological invasion success through mathematical modeling. The model will be represented by a system of differential equations with three state variables and several ecological parameters. In the first stage, the parameters that determine population trends and interspecific interactions will be constant, allowing us to know the effect of the exotic species' invasiveness and the site's invasiveness. In the second stage, the parameters that determine population trends and interspecific interactions will be functions dependent on the phenotypic traits, allowing the evaluation of the potential for phenotypic change. Finally, impulsive dynamics will be used to study the effect of propagule pressure and thus determine how the potential of phenotypic change offsets this. The model will be analyzed with underlying mathematical theories and computational tools [14, 15, 16] to later give interpretations to the results obtained. This research will provide scientific evidence that will contribute to the knowledge of biological invasions, which will serve as a basis for future control measures and management of introduced species.

References

[1] Clavero, Miguel and García-Berthou, Emili, Invasive species are a leading cause of animal extinctions, *Trends in ecology & evolution* 2005, vol. **20**, no 3, p. 110.

[2] LOCKWOOD, Julie L.; HOOPES, Martha F.; MARCHETTI, Michael P. *Invasion ecology*. John Wiley & Sons, 2013.

[3] PYŠEK, Petr, et al. Scientists' warning on invasive alien species. *Biological Reviews*, 2020, vol. **95**, no 6, p. 1511-1534.

[4] NOVAK, Stephen J. The role of evolution in the invasion process. *Proceedings of the National Academy of Sciences*, 2007, vol. **104**, no 10, p. 3671-3672.

[5] MOUGI, Akihiko. Allelopathic adaptation can cause competitive coexistence. *Theoretical Ecology*, 2013, vol. **6**, no 2, p. 165-171.

[6] BERTHON, Katherine. How do native species respond to invaders? Mechanistic and trait-based perspectives. *Biological Invasions*, 2015, vol. **17**, p. 2199-2211.

[7] MOSEBY, K. E., et al. Designer prey: can controlled predation accelerate selection for anti-predator traits in naïve populations?. *Biological Conservation*, 2018, vol. **217**, p. 213-221.

[8] CASTILLO, Maria L., et al. The contribution of phenotypic traits, their plasticity, and rapid evolution to invasion success: insights from an extraordinary natural experiment. *Ecography*, 2021, vol. **44**, no 7, p. 1035-1050.

[9] LOCKWOOD, Julie L.; CASSEY, Phillip; BLACKBURN, Tim. The role of propagule pressure in explaining species invasions. *Trends in ecology & evolution*, 2005, vol. **20**, no 5, p. 223-228.

[10] SIMBERLOFF, Daniel. The role of propagule pressure in biological invasions. *Annual Review of Ecology, Evolution, and Systematics*, 2009, vol. **40**, p. 81-102.

[11] STRINGHAM, Oliver C.; LOCKWOOD, Julie L. Managing propagule pressure to prevent invasive species establishments: Propagule size, number, and risk release curve. *Ecological Applications*, 2021, vol. **31**, no 4, p. e02314.

[12] KATS, Lee B.; FERRER, Ryan P. Alien predators and amphibian declines: review of two decades of science and the transition to conservation. *Diversity and distributions*, 2003, vol. **9**, no 2, p. 99-110.

[13] MANNA, Kalyan; BANERJEE, Malay. Stationary, non-stationary and invasive patterns for a prey-predator system with additive Allee effect in prey growth.*Ecological complexity*, 2018, vol. **36**, p. 206-217.

[14] KUZNETSOV, Yuri A.; KUZNETSOV, Iu A.; KUZNETSOV, Y. Elements of applied bifurcation theory. New York: Springer, 1998.

[15] DOEDEL, Eusebius J. Lecture notes on numerical analysis of non-linear equations. Numerical Continuation Methods for Dynamical Systems: Path following and boundary value problems, 2007, p. 1-49.

[16] GUCKENHEIMER, John; HOLMES, Philip. Nonlinear oscillations, dynamical systems, and bifurcations of vector fields. Springer Science & Business Media, 2013.

4. **Expositor:** Alejandro Rojas-Palma
Afiliación: Departmento de Matemática, Física y Estadística
Facultad de Ciencias Básicas
Universidad Católica del Maule
Talca, Chile
Título: Limit cycles and Bogdanov-Taken bifurcations of a predator-prey system
Resumen: In this work, a bidimensional continuous-time differential equations system is analyzed which is derived of Leslie-Gower type predator-prey schemes by considering a nonmonotonic functional response. For the system obtained we describe the bifurcation diagram of limit cycles that appears in the first quadrant, the only quadrant of interest for the sake of realism. We show that, under certain conditions over the parameters,

the system allows the existence of three limit cycles: The first two cycles are infinitesimal ones generated by Hopf bifurcation; the third one arises from a homoclinic bifurcation. Furthermore, we give conditions over the parameters such that the model allows long-term extinction or survival of both populations.

References

[1] Aguirre, Pablo; Gonzalez-Olivares, Eduardo; Saéz, Eduardo, *Three Limit Cycles in a Leslie-Gower Predator-Prey Model with Additive Allee Effect.*,SIAM Journal on Applied Mathematics, 69(5):1244–1262, Jan. 2009

[2] Freedman, H. I.; Wolkowicz, Gail *Predator-prey systems with group defence: The paradox of enrichment revisited.*, Bulletin of Mathematical Biology, 48(5-6):493–508, 1986.

[3] González-Olivares, Eduardo; Arancibia-Ibarra, Claudio; Rojas-Palma, Alejandro; González-Yañez, Betsabé *Bifurcations and multistability on the may-holling- tanner predation model considering alternative food for the predators*, Mathematical Biosciences and Engineering, 16(5):4274–4298, 2019.

[4] Puchuri, Liliana; González-Olivares, Eduardo; Rojas-Palma, Alejandro, *Multistability in a Leslie-Gower–type predation model with a rational nonmonotonic functional response and generalist predators*, Computational and Mathematical Methods, 2(2), Mar. 2020.