14 Calculo de Variaciones y E.D.P.

1. **Expositor:** Leonelo Iturriaga[11].
Afiliación: Universidad Técnica Federico Santa María (UTFSM).
Título: *Schrödinger equations involving weights and potentials with a sort of coercivity.*
Resumen: In this talk we establish the existence of weak solutions of the following nonlinear Schrödinger equation

$$(P) \qquad -\mathrm{div}(a(x)\nabla u) + V(x)u = \lambda f(x,u) \ \text{ in } \ \mathbf{R}^N,$$

where λ is a positive parameter, the function $f : \mathbf{R}^N \times \mathbf{R} \to \mathbf{R}$ and $a, V : \mathbf{R}^N \to (0,\infty)$ are continuous. The novelties here are: to assume that $x \mapsto a(x) + V(x)$ is coercive and, that no assumptions on the nonlinearity f at ∞ are needed. The proofs of our results are based on truncation techniques combined with a priori estimates and variational methods.

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2. **Expositor:**. Rajesh Mahadevan[15]
Afiliación: Universidad de Concepción (UDEC).
Título: *The Pólya-Szegő inequality for the Steiner symmetrization in the non-local setting and some recent applications*
Resumen: In this talk we will present the non-local Pólya-Szegő inequality for the Steiner symmetrization and discuss the equality case in the inequality. Some isoperimetric inequalities obtained as consequences of this inequality will be presented. The contents of this talk form part of the doctoral research of F. Olivares-Contador carried out under my supervision.

Joint work with:
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3. **Expositor:** Juan Carlos Pozo[17].
Afiliación: Universidad de Chile (UCH).
Título: *Decay rate for the wave equation with viscoelastic boundary damping*
Resumen: In this talk, we will show how to obtain sharp estimates on the energy decay rate for the solutions of a wave equation subject to memory damping at the boundary of the domain. Our main results can be viewed as a complement of the results obtained in [1, 2].
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4. **Expositor:** Rodolfo Viera Quezada [18].
Afiliación: Pontificia Universidad Católica de Chile (PUC).
Título: *On point configurations, distortion and PDEs*
Resumen: In 1993, Gromov [3] asked if every separated net X in the plane is bi-Lipschitz equivalent to $\mathbf{Z}^2$. This was answered in the negative by Burago and Kleiner [1], and independently by McMullen [2]. Their proof relies on the existence of a continuous function $\rho : [0,1]^2 \to \mathbf{R}$ which is bounded away from zero such that the PDE

$$Jac(f) = \rho \quad \text{a.e,} \qquad (1)$$

has not bi-Lipschitz solutions $f : [0,1]^2 \to \mathbf{R}^2$. In this talk we are interested in this interplay between problems of discrete nature, such as the (non-)existence of bi-Lipschitz rectifiable separated nets and the (non-)existence of bi-Lipschitz solutions of (1). Also we will explore some generalizations for separated nets which are non-rectifiable under a broader class of functions [4].

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5. **Expositor:** Daniel Pons.
Afiliación: Universidad Andrés Bello (UNAB).
Título: *Métricas no-canónicas en Diff(S^1)*
Resumen: Revisamos algunas ideas de V.I. Arnold sobre el grupo de difeomorfismos en variedades. Cuando la variedad subyacente es el círculo,

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estudiamos propiedades de tal grupo dotado con ciertas métricas.

Trabajo en colaboración con:
Enrique G. Reyes, Departamento de Matemáticas, USACH, Santiago, Chile.

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6. **Expositor:** Lisbeth Carrero[19].
Afiliación: NA.
Título: *Periodic fractional Ambrosetti-Prodi for one-dimensional problem*
Resumen: Inspired by the works of [3] and [4], we establish Ambrosetti-Prodi type results for periodic solutions of one-dimensional nonlinear problems with drift term and drift-less whose principal operator is the fractional Laplacian of order $s \in (0,1)$. We establish conditions for the existence and nonexistence of solutions. The proofs of the existence results are based on the sub-supersolution method combined with topological degree type arguments. We also establish a priori bounds in order to get multiplicity results. For this purpose, we will use some results of [2]. We also prove that the solutions are $C^{1,\alpha}$ under some regularity assumptions in the nonlinearities. We know that determining the regularity of solutions is, in general, a difficult subject to address and depends on the value of s. In our case, by using [5] and [6, Theorem 1.1] we get the $C^{1,\alpha}$ regularity of the solutions in the case $s > 1/2$ and $s < 1/2$ respectively. The case $s = 1/2$ is the most delicate. We did not find a regularity result in the bibliography that would be useful to us, so by using some ideas of [1] we prove the $C^{1,\alpha}$ regularity directly.

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7. **Expositor:** Mircea Petrache[22].
Afiliación: Pontificia Universidad Católica de Chile (PUC).
Título: *Reconstruction of discrete Fourier spectrum in general groups, from random perturbations*
Resumen: We consider the reconstruction problem of the Fourier spectrum data of a discrete subgroup of a general group, when the discrete subgroup is randomly perturbed. The basic result I will present is that under independence assumptions on the random perturbations, the Fourier transform of a random perturbation is close to the Fourier transform of the unperturbed subgroup, multiplied by the Fourier transform of the law of the noise. Thus for example for Gaussian i.i.d. perturbations of a lattice $X \subset \mathbb{R}^d$, the perturbed crystal's Fourier transform allows to recover the initial crystal. The case of (perturbations of) lattices is due to Yakir, and we here treat lattices in finite groups and other nilpotent groups. This is joint work with Rodolfo Viera from UC Chile [2].

Joint work with:
Rodolfo Viera[23], Departamento de Matemáticas, Universidad Católica de Chile, Santiago, Chile.

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8. **Expositor:** Carlos Román [24].
Afiliación: Pontificia Universidad Católica de Chile (PUC).
Título: *The occurrence of branch microstructures in micromagnetism*
Resumen: Nonconvex variational problems regularized by higher order terms have been used to describe many physical systems, including, for example, martensitic phase transformation, micromagnetics, and the Ginzburg–Landau model of nucleation. These problems exhibit microstructure formation, as the coefficient of the higher order term tends to zero. They can be naturally embedded in a whole family of problems of the form: minimize $E(u) = S(u) + N(u)$ over an admissible class of functions u taking only two values, say -1 and 1, with a nonlocal interaction N favoring small-scale phase oscillations, while the interfacial energy S penalizes them.
In this talk I will report on recent joint work with Tobias Ried, in which we establish self-similarity, in a statistical sense through local energy bounds, of minimizers of an energy functional that naturally arises when analyzing the behavior of uniaxial ferromagnets using the Landau–Lifschitz model.

Joint work with:
Tobias Ried[25], Max Planck Institute for Mathematics in the Sciences, Leipzig, Germany.

9. **Expositor:** Hernán Castro[26].
Afiliación: Universidad de Talca (UTAL)
Título: *Some regularity results for a degenerate quasilinear elliptic equation*
Resumen: In this talk we will discuss some recent results regarding the regularity of solutions to the quasilinear degenerate elliptic equation

$$\begin{cases} -\text{div } \mathcal{A}(x,u,\nabla u) = \mathcal{B}(x,u,\nabla u) & \text{for } x \in \Omega \subset \mathbf{R}^N, \\ u = 0 & \text{on } \partial\Omega. \end{cases}$$

where for $\mathcal{A}$, $\mathcal{B}$ are a Carathéodory functions verifying

- $\mathcal{A}(x,u,z) \sim w(x)(|\nabla u|^{p-2}\nabla u + |u|^{p-2}u + f(x))$.

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- $\mathcal{B}(x, u, z) \sim w(x)(|\nabla u|^{p-2}\nabla u + |u|^{p-2}u + g(x))$.

where w is a weight function satisfying suitable conditions.

We are particularly interested in the case $w(x) = x^A$ where for $A = (a_1, a_2, \ldots, a_N) \in (\mathbf{R}_+)^N$ the function x^A is the monomial weight

$$x^A = |x_1|^{a_1} \cdot |x_2|^{a_2} \cdot \ldots \cdot |x_N|^{a_N}$$

and for positive solutions to the problem where $\mathcal{A}(x, u, z) = x^A|z|^{p-2}z$, $\mathcal{B}(x, u, z) = x^A|u|^{r-1}r$ in an open set $\Omega \subseteq (\mathbf{R}_+)^N$.

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10. **Expositor:** Renjith Thazhathethil [27].
Afiliación: Indian Institute of Science (Bangalore, India).
Título: *Homogenization with strong contrasting diffusivity in a circular oscillating domain with L^1 source term*
Resumen: In this talk, we present a novel approach to the homogenization of an elliptic partial differential equation (PDE) with highly oscillating coefficients in a circular domain, Our focus is on the combination of three challenging factors: an oscillatory boundary, rapid oscillating coefficients, and the presence of two materials with high contrasting conductivity (or diffusivity) with a source term in L^1. The resulting non-uniform ellipticity as the oscillating parameter goes to zero requires a unique approach, which we provide through our results. The use of a sourse term in L^1 leads to solutions that are interpreted as renormalized solutions, requiring a careful analysis.

We begin by proving strong convergence results (corrector results) for the PDE with an L^2 source term. This serves as a foundation for our homogenization results for the renormalized form. We establish the relationship between the ε-stage renormalized solution and the limit renormalized solution via convergence results. Our analysis relies on the use of the unfolding operator for polar coordinates, which plays a central role in our work.

The results presented in this talk are published in [1].

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Joint work with:
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11. **Expositor:** Hanne Van Den Bosch.
Afiliación: Universidad de Chile (DIM-UCH).
Título: *A Keller-Lieb-Thirring Inequality for Dirac operators*
Resumen: The *classical* Keller-Lieb-Thirring inequality bounds the ground state energy of a Schrödinger operator by a Lebesgue norm of the potential. This problem can be rewritten as a minimization problem for the Rayleigh quotient over both the eigenfunction and the potential. It is then straightforward to see that the best potential is a power of the eigenfunction, and the optimal eigenfunction satisfies a nonlinear Schrödinger equation. This talk concerns the analogous question for the smallest eigenvalue in the gap of a massive Dirac operator. This eigenvalue is not characterized by a minimization problem. By using a suitable Birman-Schwinger operator, we show that for sufficiently small potentials in Lebesgue spaces, an optimal potential and eigenfunction exists. Moreover, the corresponding eigenfunction solves a nonlinear Dirac equation.

Joint work with: Jean Dolbeault, David Gontier and Fabio Pizzichillo.

12. **Expositor:** Duvan Henao[30].
Afiliación: Universidad de O'Higgins (UOH).
Título: *Desprendimiento de hidrogeles delgados*
Resumen: Motivados por el problema de desarrollar los polímeros sintéticos con los que se recubren marcapasos y otras prótesis médicas, ponemos en un marco unificado el modelo entrópico de Flory y Rehner para la absorción de fluido por parte de un polímero (uno de los modelos que hizo a Flory merecedor del Nobel en Química en 1974); el análisis asintótico para modelos reducidos de películas delgadas; y la formulación variacional de la mecánica de fracturas. Este trabajo, en el que experimentos se contrastan con las simulaciones en elementos finitos y las conclusiones del análisis matemático, permite estimar la tasa de energía liberada a medida que el

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hidrogel se desprende de un sustrato rígido plano. En un problema bidimensional simplificado, se obtiene una cota inferior para el grosor a partir del cual el revestimiento de hidrogel se desprende en forma espontánea.

Trabajo realizado junto a:
M. Carme Calderer[31], **C. Garavito**[32],**S. Lyu**[33],
M. A. Sánchez[34], **R. A. Siegel**[35],**S. Song**[36],**L. Tapia**[37]

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13. **Expositor:** Nicolás Valenzuela[38].
Afiliación: Universidad de Chile (DIM-UCH).
Título: *Una nueva visión para el Laplaciano fraccionario via redes neuronales profundas.*
Resumen: Esta charla se enfocará en el estudio del problema de Dirichlet con Laplaciano fraccionario y su relación con redes neuronales profundas. En particular, a partir de una representación estocástica de la solución del problema de Dirichlet fraccionario dado por [1], demostraremos que existe una red neuronal profunda que aproxima la solución del problema a una precisión arbitraria y más aún, la cantidad de parámetros necesaria para definir la red es a lo más polinomial en la dimensión, el recíproco de la precisión y el tamaño del conjunto [2]. Además se realizarán simulaciones numéricas en Python para encontrar redes neuronales que aproximan las soluciones del problema de Dirichlet fraccionario via Gradiente Descenso Estocástico.

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14. **Expositor:** Carlo Santana Rosas[39]
Afiliación: Universidad Nacional de Ingeniería UNI (Lima, Perú)
Título: *Condiciones de optimalidad para una clase de problemas de cálculo de variaciones en horizonte infinito*
Resumen: En [1] se demuestra que las curvas del tipo MRAP's (Most rapid approach paths) que son del tipo bang-bang o bang-singular, son soluciones óptimas de una clase de problemas de cálculo de variaciones en horizonte infinito, lineal con respecto a la variable de velocidad, sujeto a un conjunto admisible formado por funciones reales definidas en $\mathbf{R}_+$, soluciones de una inclusión diferencial.
Lo que estamos pretendiendo ahora es estudiar este problema en dimensión superior (i.e., cuando los elementos del conjunto admisible son funciones que toman valores en $\mathbf{R}^n$ definidas en $\mathbf{R}_+$). Algunas dificultades encontradas en esta investigación es definir de forma adecuada las curvas MRAPs en esta versión más general y dar alguna condición necesaria de optimalidad que no hay en la literatura. Para ello, transformamos nuestro problema de cálculo de variaciones a un problema de control óptimo, los cuales estos dos problemas resultan ser equivalentes en algún sentido. Luego, usamos las herramientas de control óptimo como el Principio del Máximo de Pontryagin [2], con el fin de estudiar las soluciones óptimas de nuestro problema, surgieno ideas para obtener una condición necesaria de optimalidad de nuestro problema variacional.

Joint work with:
Eladio Ocaña Anaya[40], IMCA, UNI, Lima, Perú.

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