1 Geometría

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An upper bound for hypersurfaces without lines in finite projective spaces

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Abstract

Let X be a hypersurface in $\mathbb{P}^{n+1}$ with $n \geq 1$ defined over a finite field $\mathbb{F}_q$ of q elements. In this talk, we prove an upper bound on the number of $\mathbb{F}_q$-points for hypersurface X as above that do not contain lines and we classify, up to projective equivalence, those X that reach this bound.

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On quasiplatonic Riemann surfaces with complex multiplication

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Abstract

In this talk we show that, for each odd integer $n \geq 3$, the unique quasiplatonic Riemann surface of genus $2(n-1)$ admitting the action of a semidirect product $\mathbb{Z}_2^2 \rtimes \mathbb{Z}_{2n}$ has complex multiplication.

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On pseudo-symmetric p-hyperelliptic Riemann surfaces

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Abstract

A compact Riemann surface of genus $g \geq 2$ is called *pseudo-symmetric* [1, 3], if it admits an anticonformal automorphism of order 4, but no anticonformal involutions. In this talk, we study pseudo-symmetric Riemann surface of genus $g \geq 2$ with full automorphism group

$$Q_{4n} = \langle x, y : \; x^{2n} = 1, y^2 = x^n, yxy = x^{-1} \rangle, \text{ for } n \text{ prime.}$$

We give necessary and sufficient conditions for the existence of such a surface and we find all values of p and q for which the surface is p-hypereliptic and (q, n)-gonal [2].

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Regular dessins d'enfants with field of moduli $\mathbb{Q}(\sqrt[p]{2})$

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Abstract

A dessin d'enfant (or hypermap) of genus g, as defined by Grothendick in his Esquisse d'un Programme [7], is a bipartite map (vertices come in black and white colors and vertices of the same color are non-adjacent) on a closed orientable surface of genus g. The degree of the dessin d'enfant is the number of its edges. As a consequence of the classical uniformization theorem, a dessin d'enfant can also be seen as a pair (S, β), where S is a closed Riemann surface and $\beta : S \to \widehat{\mathbb{C}}$ is a non-constant meromorphic map whose branch values are contained in the set $\{\infty, 0, 1\}$; the degree of the dessin is the same as the degree of β. A dessin d'enfant (S, β) is called regular if β is a regular branched covering. The signature of the dessin d'enfant is the tripe (a, b, c), where a (respectively, b and c) is the least common multiple of the local degrees of β at each preimage of 0 (respectively, 1 and ∞). In terms of the bipartite map, a is the least common multiple of the degrees of black vertices, b is the least common multiple of the degrees of white vertices and c is the least common multiple of the degrees of the faces (recall that a face of the dessin d'enfant must have an even number 2δ of boundary edges, where internal edges are counted twice; in this case δ is the degree of the face). Two dessins d'enfant (S_1, β_1) and (S_2, β_2) are said to be equivalent (denoted this by the symbol $(S_1, \beta_1) \sim (S_2, \beta_2)$) if there is an isomorphism $f : S_1 \to S_2$ so that $\beta_1 = \beta_2 \circ f$. Clearly, the signature is an invariant under this equivalence relation. There is a natural bijection between dessins d'enfants (respectively, regular dessins d'enfants), of signature (a, b, c) and degree d, and conjugacy classes of subgroups (respectively, normal subgroups) of index d of the triangular group $\Delta(a, b, c) = \langle x, y : y^a = x^b = (xy)^c = 1\rangle$. By Belyi's theorem [2], each dessin d'enfant is equivalent to a dessin d'enfant (C, β) where C is an algebraic curve and β a rational map, both defined over the field $\overline{\mathbb{Q}}$ of algebraic numbers. This provides a natural action of the absolute Galois group $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ on the set of (equivalence classes of) dessins d'enfants as follows. Start with a dessin d'enfant (C, β), defined algebraically over $\overline{\mathbb{Q}}$ and let $\sigma \in \mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$. Assume C is defined by the polynomials $P_1, \ldots, P_r$ and that $\beta = Q_1/Q_2$, where all polynomials have coefficients in $\overline{\mathbb{Q}}$. Let P_j^σ and Q_k^σ be the polynomials obtained by applying σ to the coefficients of P_j and Q_k, respectively. If C^σ is the algebraic curve defined by the polynomials P_j^σ and $\beta^\sigma = Q_1^\sigma/Q_2^\sigma$, then (C^σ, β^σ) is still a dessin d'enfant. It is well known that the above action of the absolute Galois group is faithful [4, 5, 7, 11]. For many years, it was an open and difficult question if the absolute Galois group also acts faithfuly on the set of regular dessins d'enfants. Last year, this problem was solved by González-Diez and Jaikin-Zapirain in [6] and in a slightly weaker form by Bauer, Catanese and Grunewald in [1]. The field of moduli of a dessin d'enfant (C, β) is the fixed field of the subgroup of $\mathrm{Gal}(\overline{\mathbb{Q}}/\mathbb{Q})$ consisting of those σ for which $(C^\sigma, \beta^\sigma) \sim (C, \beta)$ (i.e., the field of definition of the equivalence class of the dessin d'enfant). The field of moduli is contained in any field of definition of the dessin (it is in fact the intersection of all of them by results due to Koizumi [10]), but there are examples for which the field of moduli is not a field of definition of it. In [12], Wolfart observed that regular dessins d'enfants are definable over its field of moduli. The only explicit examples for such Galois Belyi actions were however known only for curves and dessins defined over abelian extensions of $\mathbb{Q}$. A question posed by Conder, Jones, Streit and Wolfart in [3] was if there were examples of regular dessins d'enfant with field of moduli being

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a non-abelian extension of $\mathbb{Q}$. In [8] Herradon answered the above positively by constructing a regular dessin d'enfant with field of moduli being $\mathbb{Q}(\sqrt[3]{2})$. Herradon starts with the following genus one non-uniform dessin d'enfant of signature $(4,6,12)$

$$\left(C : y^2 = x(x-1)\left(x-\sqrt[3]{2}\right), \quad \beta(x,y) = x^3(2-x^3)\right),$$

whose field of moduli is $\mathbb{Q}(\sqrt[3]{2})$, and then he observes that its normalizing regular dessin d'enfant has the same field of moduli (he also constructs another regular dessin d'enfant with the same property, this being a quotient of the previous one). In this talk we present an infinite family of regular dessins d'enfants whose field of moduli is $\mathbb{Q}(\sqrt[p]{2})$, where p runs over the prime integers. Herradon's example belongs to our family for $p=3$.

Joint work with:
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Application of algebraic geometry to calculation of Feynman diagrams

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Abstract

In Refs.[1, 2, 3, 4] a family of ladder Feynman diagrams in quantum field theory has been studied for integer and non-integer space-time dimensions. We represent these results in terms of different families of polylogarithms, for example, in terms of Grassmannian n-logarithms.

Joint work with:
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Períodos de curvas generalizadas de Fermat.

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Abstract

Una curva generalizada de Fermat de tipo (k, n) es una superficie de Riemann compacta que admite un subgrupo de automorfismos isomorfo a $\mathbb{Z}_k^n$. En [1] los autores prueban que toda curva generalizada de Fermat es conformalmente equivalente a un producto fibrado de curvas de Fermat clásicas. Para las curvas de Fermat clásicas, los autores en [2] exhiben un método para construir un conjunto de generadores para el primer grupo de homología y que son utilizados para calcular los períodos sobre dichos generadores. En esta charla se mostrará como es posible generalizar esta idea para las curvas de generalizadas de Fermat. Los resultados expuestos son parte mi trabajo de tesis.

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Resolución simultanea incrustada no implica deformación $\mu^\star$-constante

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Abstract

Sea $\mathbb{K}$ un cuerpo algebraicamente cerrado de característica cero, $V \subset \mathbb{A}^{n+1}_{\mathbb{K},0}$ un germen de hipersuperficie con una singularidad aislada en el origen y $\pi : W \to S := \mathrm{Spec}(\mathbb{K}[s])_0$ una deformación incrustada de V (es decir, W es una hipersuperficie de $\mathbb{A}^{n+1}_S := \mathbb{A}^{n+1}_{\mathbb{K},0} \times S$, π es un morfismo plano y la fibra esquemática $W_0 = W \times_S \{0\}$ es isomorfa a V).

Diremos que W admite una *resolución simultanea incrustada*, si existe un morfismo $\rho : \widetilde{\mathbb{A}}^{n+1}_S \to \mathbb{A}^{n+1}_S$ propio relativo a S que satisface: $\widetilde{\mathbb{A}}^{n+1}_S$ es suave relativo a S; $\rho : \widetilde{W}^e \to W$ es una resolución simultanea muy débil (es decir, es una resolución de la singularidad fibra a fibra); $\widetilde{W}^t$ es un divisor con cruzamientos normales relativo a S. Donde $\widetilde{W}^e$ y $\widetilde{W}^t$ es la transformada estricta y la transformada total de W por ρ, respectivamente.

El entero $\mu^i(V)$ es el número de milnor de $V \cap H$, donde H es un i-plano genérico de $\mathbb{A}^{n+1}_{\mathbb{K},0}$, $1 \leq i \leq n+1$. Diremos que la deformación incrustada $\pi : W \to S := \mathrm{Spec}(\mathbb{K}[s])_0$ es una *deformación $\mu^\star$-constante* si para todo $1 \leq i \leq n+1$ y $s' \in S$ se tiene que $\mu^i(W \times_S \{s'\}) = \mu^i(V)$.

En el articulo [1] se afirma que si la deformación incrustada $\pi : W \to S$ admite una resolución simultanea incrustada, entonces π es una deformación $\mu^\star$-constante. En esta charla presentaremos un contra ejemplo a esta afirmación y daremos algunas consecuencias a nivel de la familia de espacios de m-jets asociada a π.

Este resultado es una parte de un trabajo en desarrollo en conjunto con:
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Computing Riemann matrices using group actions.

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Abstract

Let A be a principally polarized abelian variety with the action of a finite group G. The decomposition of the semisimple algebra $\mathbb{Q}[G]$ into simple algebras induces the isotypical decomposition of A. Furthermore, the decomposition of a simple algebra into minimal left ideals, induces a finer decomposition of A into primitive factors, since they are image of primitive idempotents in $\mathbb{Q}[G]$, although they are not simple (as abelian varieties) in general. This is called the group algebra decomposition of A, and it is as far as A can be decomposed using the group action.

In this work we present a method to compute the Riemann matrices of the isotypical factors decomposing the abelian variety A, without knowing the Riemann matrix Z_A of A, provided a symplectic representation for the action of G on A. This is done by reversing the method in [2] to find Z_A from the Riemann matrices of the isotypical factors. Furthermore, once having the period matrices of the isotypical factors, we find the Riemann matrices of the primitive factors, by a minor adjustment of [2]. Finally we show an example of a Jacobian variety for which it is computationally very expensive to compute its Riemann matrix directly using the group action on it, the method proposed here solves the problem easily. Moreover, we decompose the primitive factors even further in this case, using [1].

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