1 Análisis Numérico

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Contents

A posteriori error estimates for the Stokes problem with singular sources

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Abstract

We propose a posteriori error estimators for classical low–order inf–sup stable and stabilized finite element approximations of the Stokes problem with singular sources in two and three dimensional Lipschitz, but not necessarily convex, polytopal domains. The designed error estimators are proven to be reliable and locally efficient. On the basis of these estimators we design a simple adaptive strategy that yields optimal rates of convergence for the numerical examples that we perform.

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Mathematical and numerical analysis for advection-reaction-diffusion-Brinkman systems

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Abstract

We are interested in modelling the interaction of bacteria and certain nutrient concentration within a porous medium admitting viscous flow. The governing equations consist of an advection-reaction-diffusion system representing the bacteria-chemical mass exchange, coupled to the Brinkman problem written in terms of fluid vorticity, velocity and pressure, and describing the flow patterns driven by an external source depending on the local distribution of the chemical species. A priori stability bounds are derived for the uncoupled problems, and the solvability of the full system is analysed using a fixed-point approach. We introduce a primal-mixed finite element method to numerically solve the model equations, employing a primal scheme with piecewise linear approximation of the reaction-diffusion unknowns, while the discrete flow problem uses a mixed approach based on Raviart-Thomas elements for velocity, Nédélec elements for vorticity, and piecewise constant pressure approximations. In particular, this choice produces exactly divergence-free velocity approximations. We establish existence of discrete solutions and show their convergence to the weak solution of the continuous coupled problem. Finally, we report several numerical experiments illustrating the behaviour of the proposed scheme.

Joint work with:
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Finite element analysis of a stabilised nonconforming scheme applied to Lamé equations

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Abstract

It is well known that the convergence rate for the standard displacement method for linear elasticity problem, using continuous linear finite elements deteriorates as the Lamé constants becomes large, i.e., when the elastic material is nearly incompressible. This difficulty can be circumvented via mixed methods based on the Hellinger-Reissner principle, that is, schemes based on the simultaneous approximation of the stress and displacement. In particular, there are modifications of the Hellinger-Reissner functional, in which the symmetry of the stress tensor is enforced weakly in order to ensure simple stable pairs for the stability condition. This option is generally achieved via the introduction of additional unknown which implies an increasing of the degrees of freedom in the final system.

Alternatively, a simple mixed formulation preserving locking free for elasticity problem is studied in Section VI.1 of [1], which search to approximate an additional scalar unknown, called pressure, and the displacements. This formulation looks more cheap than the ones based on Hellinger-Reissner functional, however, it keeps the need to satisfy a discrete inf–sup condition, which implies that it does not available to use any pair of finite element spaces. In particular, for lowest order nonconforming Crouzeix–Raviart element, which, up to the author's knowledge, is the simplest finite element that satisfies the discrete inf–sup stability condition with the piecewise constant pressure space. The main hindrance is that the lowest order nonconforming approach does not satisfy Korn's inequality in the discrete setting (see [3, 2]). Here we present an a-priori error analysis of an stabilisation allowing to circumvent this difficulty

Joint work with:
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An a priori error analysis for discontinuous Lagrangian finite element method for Stokes problem

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Abstract

In this talk we extend the analysis described in a previous work ([2]), and apply it to study a discontinuous mixed formulation for Stokes problems, considering only piecewise discontinuous polynomial approximations for each unknown. One important consequence of the analysis described here is the fact that we can prove that piecewise polynomial ($\mathbf{P}_r$) approximations of functions in $H(div)$ behave as good as when one deals with the usual Raviart-Thomas (RT_r) approximation ones (as in [2]). It is relevant to notice that Raviart-Thomas elements do satisfy the Rham's commutative diagram, which is not our situation, since we are using piecewise Lagrange shape functions for approximation. Numerical examples are included and confirm the theoretical results of this approach.

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*Partially supported by CONICYT-Chile, through the FONDECYT project No. 1160578 and BASAL project (PFB03), CMM, Universidad de Chile; by Centro de Investigación en Ingeniería Matemática (CI^{2}MA), Universidad de Concepción;. e-mail: `rbustinz@ing-mat.udec.cl`
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A dPG framework for strongly monotone operators

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Abstract

We present and analyze a hybrid technique to numerically solve strongly monotone nonlinear problems by the discontinuous Petrov–Galerkin method using optimal test functions. Our strategy is to relax the nonlinear problem to a linear one with additional unknowns and to consider the nonlinear relation as a constraint. We propose to use optimal test functions only for the linear problem and to enforce the nonlinear constraint by penalization. Our scheme can be seen as a minimum residual method with nonlinear penalty term. We prove under appropriate assumptions the well-posedness of the continuous formulation and the quasi-optimal convergence of its discretization. As an application we consider an advection-diffusion problem with nonlinear diffusion of strongly monotone type. Some numerical results in the lowest-order setting are presented to illustrate the predicted convergence.

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Hybridizible Discontinuous Galerkin method for linear elasticity in curved domains

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Abstract

This work proposes a hybridize discontinuous Galerkin (HDG) method for the linear elasticity problem in domains Ω that are not necessarily polyhedral/polygonal. In particular, we approximate the domain by a polyhedral/polygonal computational domain D_h where the HDG solution can be computed. The Dirichlet boundary data is suitable transferred from the boundary $\Gamma := \partial\Omega$ to the computational boundary $\Gamma_h := \partial D_h$. We show that the scheme is well-posed. Moreover, we prove a priori error estimates showing that the method is optimal. In addition, we prove that the numerical trace is superconvergent with order $k+2$ if the distance between Γ and Γ_h is of order h^2. On the other hand, if this distance is of order h, then the numerical trace superconvergences with rate $k+3/2$. We validate our theoretical results with numerical experiments in two-dimension.

Joint work with:
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On a mixed method for buoyancy driven double diffusive convection

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Abstract

Natural convection is of paramount interest since it appears in many environmental processes and diverse industrial practical applications. In this talk, I will present a work in progress that extends a previous mixed finite element method for heat driven flows to the more complex and realistic situation in which this phenomena results from bouyancy forces generated not only by temperature differences but also species concentration effects. The model in consideraton is written in the Oberbeck-Boussinesq approximation framework; a system given by the incompressible Navier-Stokes equations, and the advection-difussion equations for describing the underlying hydrodynamic, and the substance concentration and the temperature, respectively. The main features of the approach and the continuous and discrete solvability results obtained so far for the corresponding variational formulation will be presented along with some preliminary numerical experiments suggesting optimal a priori error estimates.

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A Least-Squares Method for the Obstacle Problem

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Abstract

En esta charla presentamos resultados recientes sobre un método del tipo mínimos cuadrados para el problema del obstáculo. Este problema consta en hallar la posición del equilibrio de una membrana forzada a estar sobre un obstáculo dado. Discutimos el problema y su reformulación a un sistema del primer orden. Eso se puede reescribir como un problema de cuadrados mínimos introduciendo una nueva variable que se puede interpretar como fuerza de reacción del obstáculo. Una dificultad que nos enfrenta es que la solución tiene en general una regularidad reducida. Por eso introducimos y analizamos estimadores a posteriori. Finalmente presentamos algunos ejemplos para demostrar la eficiencia de nuestro método propuesto.

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Adaptive finite element methods for sparse optimal control problems

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Abstract

We propose and analyze reliable and efficient a posteriori error estimators for an optimal control problem that involves a nondifferentiable cost functional, the Poisson problem as state equation and control constraints. To approximate the solutions to the state and adjoint equations we consider piecewise linear discretizations whereas two different strategies are used to approximate the control variable: piecewise constant discretization and the so–called variational discretization approach. For the first aforementioned solution technique we devise an error estimator that can be decomposed as the sum of four contributions: two contributions that account for the discretization of the control variable and the associated subgradient, and two contributions related to the discretization of the state and adjoint equations. The error estimator for the variational discretization approach is decomposed only in two contributions that are related to the discretization of the state and adjoint equations. On the basis of the devised a posteriori error estimators, we design simple adaptive strategies that yield optimal rates of convergence for the numerical examples that we perform.

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Análisis de un método de elementos finitos mixtos conservativo para el problema de Navier-Stokes estacionario.

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Abstract

En este trabajo, proponemos y analizamos un nuevo método de elementos finitos mixtos para el problema de Navier-Stokes estacionario usando espacios de Banach no usuales. Nuestro enfoque se basa en la introducción de un tensor no-lineal de "pseudoesfuerzos" el cual incorpora un término convectivo, con lo que se obtiene una formulación mixta con el tensor de "pseudoesfuerzos" indicado anteriormente y la velocidad como las principales incógnitas del sistema. Hay variables adicionales de interés, tales como la presión, vorticidad y el gradiente de la velocidad del fluido, que pueden ser fácilmente aproximadas por medio de un post-proceso y que mantienen la misma razón de convergencia que las incógnitas principales. Los teoremas de Banach–Nečas–Babuška y el del punto fijo de Banach son las principales herramientas para probar el buen planteamiento del problema continuo. Similarmente, establecemos el buen planteamiento del esquema de Galerkin asociado, junto con la correspondiente estimación de Cea considerando espacios de elementos finitos conformes para cada incógnita. En particular, el esquema de Galerkin asociado puede ser definido usando elementos de Raviart-Thomas de grado k para el tensor de pseudoesfuerzo y funciones continuas polinomiales a trozos de grado $k+1$ para la velocidad, lo cual permite obtener un esquema optimalmente convergente. Además, proporcionamos dos métodos iterativos para resolver el correspondiente sistema de ecuaciones no-lineal y analizar su convergencia. Finalmente, diversos ensayos numéricos ilustran el buen rendimiento del método.

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Some remarks about ultraweak formulations of the Kirchhoff–Love plate bending problem

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Abstract

In [1] we have developed and analyzed an ultraweak variational formulation for a variant of the Kirchhoff–Love plate bending model. Based on this formulation, we introduced a discretization of the discontinuous Petrov–Galerkin type with optimal test functions (DPG) and proved the well-posedness of the ultraweak formulation and quasi-optimal convergence of the DPG scheme.

Essential for the analysis is the appropriate treatment of inherent traces, jumps and trace spaces. An essential difficulty stems from the fact that the bending tensor variable has a twice-iterated divergence in L_2 but that its (single) divergence is less regular in general.

In this talk we will discuss this problem and also deal with an extended formulation that involves the so-called rotation (or gradient of the deflection) as an independent unknown.

Joint work with:
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Polynomial approximation of high-dimensional Hamilton-Jacobi-Bellman equations

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Abstract

In this talk we present a procedure for the numerical approximation of high-dimensional Hamilton-Jacobi-Bellman (HJB) equations associated to optimal feedback control problems for semilinear parabolic equations [1]. Its main ingredients are a pseudospectral collocation approximation of the PDE dynamics, and an iterative method for the nonlinear HJB equation associated to the feedback synthesis [2]. The latter is known as the Successive Galerkin Approximation. It can also be interpreted as Newton iteration for the HJB equation. At every step, the associated linear Generalized HJB equation is approximated via a separable polynomial approximation ansatz. We obtain stabilizing feedback controllers from solutions to the HJB equation for systems of dimension up to fourteen.

Joint work with:
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Modelling flotation columns through a conservation law with multiply discontinuous flux

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Abstract

Flotation is a unit operation that is extensively used in the recovery of valuable minerals and coals in mineral processing and related applications. The theory of froth flotation is complex, involving three phases (solids, water, and froth or gas) with many subprocesses. However, essential insight to the hydrodynamics of a flotation column can be obtained by studying just two phases: gas and fluid [2, 3]. In [1], the authors proposed a reformulation of the approach based on the drift-flux theory, as a one-dimensional non-linear conservation law with a multiply discontinuous flux: the unknown is the gas volume fraction as a function of height and time, and the flux function depends discontinuously on spatial position due to feed inlets for gas, feed slurry and wash water. The resulting model adds a new real-world application to the field of conservation laws with discontinuous flux. Steady-state solutions were studied in detail, including their construction by applying appropriate entropy conditions across each flux discontinuity. This analysis leads to operating charts and tables collecting all possible steady states along with some necessary conditions for their feasibility in each case. Numerical experiments showed that the transient model recovers the steady states, depending on the feed rates of the different inlets.

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Esquema numérico de segundo orden para Leyes de Conservación con flujo restringido

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Abstract

Los problemas de tráfico vehicular con restricción en un punto pueden ser modelados por leyes de conservación con restricción de flujo, una de las ecuaciones diferenciales parciales tipo hiperbólicas que lo representa es

$$\begin{cases} \partial_t \rho + \partial_x f(\rho) = 0, & t > 0,\ x \in \mathbb{R} \\ \rho(0, x) = \rho_0(x), & x \in \mathbb{R} \\ f(\rho(t, 0)) \leq F(t), & t > 0. \end{cases} \tag{1}$$

El buen planeamiento de este tipo de problemas puede ser llevado a cabo a través de esquemas de primer orden. En esta charla se propone un esquema de segundo orden modificado en los puntos donde actúa la restricción, para que el esquema aproxime mejor a la solución del problema.

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*Parcialmente financiado por proyecto FONDECYT 11140708, e-mail: ycmartinez@ubiobio.cl
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Virtual Elements for a Shear-Deflection Formulation of Reissner-Mindlin Plates

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Abstract

We present a virtual element method for the Reissner–Mindlin plate bending problem which uses shear strain and deflection as discrete variables without the need of any reduction operator. The proposed method is conforming in $[H^1(\Omega)]^2 \times H^2(\Omega)$ and has the advantages of using general polygonal meshes and yielding a direct approximation of the shear strains. The rotations are then obtained by a simple postprocess from the shear strain and deflection. We prove convergence estimates with involved constants that are uniform in the thickness t of the plate. Finally, we report numerical experiments which allow us to assess the performance of the method.

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A stabilised finite element method for the convection–diffusion–reaction equation in mixed form

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Abstract

This work is devoted to the approximation of the convection–diffusion–reaction equation using a mixed, first order, formulation. We propose, and analyse, a stabilised finite element method that allows equal order interpolations for the primal and dual variables. This formulation, reminiscent of the Galerkin least-squares method, is proven stable and convergent. In addition, a numerical assessment of the numerical performance of different stabilised finite element methods for the mixed formulation is carried out, and the different methods are compared in terms of accuracy, stability, and sharpness of the layers for two different classical test problems.

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*This research was partially supported by Dirección de Investigación e Innovación, Universidad Católica de la Santísima Concepción through project DINREG 04/2017, e-mail: apoza@ucsc.cl
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Estimaciones de error a posteriori para un problema de control óptimo de la ecuación de Stokes

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Abstract

Sea $\Omega \subset \mathbb{R}^d$, con $d \in \{2,3\}$, un dominio abierto y acotado, cuya frontera $\partial\Omega$ es Lipschitz continua con forma poligonal. Consideremos el siguiente problema de control óptimo:

$$\min \ J(\mathbf{y},\mathbf{u}) := \frac{1}{2}\|\mathbf{y}-\mathbf{y}_\Omega\|^2_{\mathbf{L}^2(\Omega)} + \frac{\alpha}{2}\|\mathbf{u}\|^2_{\mathbf{L}^2(\Omega)}, \tag{1}$$

sujeto a la ecuación de Stokes

$$\left\{\begin{array}{rcll} -\nu\Delta\mathbf{y}+\nabla p &=& \mathbf{f}+\mathbf{u}, & \text{en } \Omega,\\ \nabla\cdot\mathbf{y} &=& 0, & \text{en } \Omega,\\ \mathbf{y} &=& \mathbf{0}, & \text{en } \partial\Omega, \end{array}\right. \tag{2}$$

y a la restricción de caja

$$\mathbf{u}_a \le \mathbf{u} \le \mathbf{u}_b \quad \text{c.t.p. en } \Omega, \tag{3}$$

donde $\mathbf{f},\mathbf{y}_\Omega \in \left[L^2(\Omega)\right]^d$ son funciones dadas, α,ν son parámetros positivos y $\mathbf{u_a},\mathbf{u_b} \in \mathbb{R}^d$ son vectores tales que $\mathbf{u_a} < \mathbf{u_b}$. En este caso las desigualdades de vectores mencionadas deben entenderse componente a componente.

El propósito de este trabajo es construir y analizar estimadores de error a posteriori para distintos esquemas de elementos finitos que permiten discretizar el problema (1) - (3). En esta charla se comenzará presentando el problema débil de Stokes, junto con distintos esquemas de discretización por elementos finitos (inf - sup estables y estabilizados de bajo orden), para luego presentar el problema de control óptimo y algunos resultados importantes. Posteriormente se exhibirán los pasos que permiten obtener estimadores de error a posteriori, que satisfagan propiedades de confiabilidad y eficiencia, para el error de discretización del problema (1) - (3). Para finalizar se pretende mostrar algunos resultados numéricos que permitan medir el desempeño de los estimadores de error obtenidos, utilizando un método de resolución adaptativo, basado en los estimadores, y un algoritmo de resolución para el problema de optimización.

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A priori and a posteriori error estimates for a virtual element spectral analysis for the elasticity equations

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Abstract

This work present a priori and a posteriori error analysis of a Virtual Element Method (VEM) to approximate the vibration frequencies and modes of an elastic solid. We propose a variational formulation of the spectral problem relying only on the solid displacement an $H^1(\Omega)$-conforming discretization by means of VEM, which is based on [1] in order to construct a proper L^2-projection operator, which is used to approximate the bilinear form on the right hand side of the spectral problem. Under standard assumptions on the polygonal meshes, we establish that the resulting VEM scheme provides a correct approximation of the spectrum and prove optimal order error estimates for the eigenfunctions and a double order for the eigenvalues. Since, the VEM has the advantage of using general polygonal meshes, which allows implementing efficiently mesh refinement strategies, we also introduce a residual-type a posteriori error estimator and prove its reliability and efficiency. We use the corresponding error estimator to drive an adaptive scheme. Finally, we report the results of a couple of numerical tests that allow us to assess the performance of this approach.

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Numerical methods for a High order Nonlinear Schrödinger Equation modelling Fiber Optics

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Abstract

In this work we present a Finite Difference scheme used to solve a High order Nonlinear Schrödinger Equation. These equations can model the propagation of solitons travelling in fiber optics, and the behavior of the envelope of the electric field in order to obtain the Kerr effect. The scheme is designed to preserve the numerical L^2 norm and the energy, for a suitable initial condition. We show numerical results displaying conservation properties of the schemes.

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Dispersion analysis of HDG methods

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Abstract

This work presents a dispersion analysis of the Hybrid Discontinuous Galerkin (HDG) method. Considering the Helmholtz system, we quantify the discrepancies between the exact and discrete wavenumbers. In particular, we obtain an analytic expansion for the wavenumber error for the lowest order Single Face HDG (SFH) method. The expansion shows that the SFH method exhibits convergence rates of the wavenumber errors comparable to that of the mixed hybrid Raviart-Thomas method. In addition, we observe the same behavior for the higher order cases in numerical experiments.

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A posteriori error analysis of an HDG method for the Oseen problem

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Abstract

In this talk, we will introduce and analyze an *a posteriori* error estimator for a hybridizable discontinuous Galerkin method applied to the gradient-velocity-pressure formulation of the Oseen problem. We establish reliability and local efficiency results of our estimator for the L^2-error of the velocity gradient and the pressure and the H^1-error of the velocity. Finally, we provide some numerical experiments showing the quality of our adaptive scheme.

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A Virtual Element Method for Thin Plates

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Abstract

In this talk, we develop a virtual element method (VEM) for the vibration problem of thin plates on polygonal meshes. We consider a variational formulation relying only on the transverse displacement of the plate and propose an $H^2(\Omega)$ conforming discretization by means of the VEM which is simple in terms of degrees of freedom and coding aspects. Under standard assumptions on the computational domain, we establish that the resulting scheme provides a correct approximation of the spectrum and prove optimal order error estimates for the eigenfunctions and a double order for the eigenvalues. Finally, we report several numerical experiments illustrating the behaviour of the proposed scheme and confirming our theoretical results on different families of meshes.

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XXXI Jornada de Matemática de la Zona Sur
Universidad Austral de Chile
25, 26 y 27 de abril de 2018, Valdivia, Chile

Discretization of a linear viscoelastic Timoshenko beam using reduced integration techniques

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Abstract

In recent years, viscoelastic materials has gained importance in the engineering industry due to their ability to show both, elastic solid and viscous fluid properties. The theory behind this materials is well established and we refer to some remarkable studies such as those from Flugge [1] or Christensen [2] for a deep analysis, numerical methods and solution techniques.
When working with small displacements or linear materials, it is possible to apply the infinitesimal stress theory through the Boltzmann superposition principle. Thus, we can find the constitutive equations integrally or use the mechanical analogs such as the Standard Linear Solid, Maxwell, Kelvin-Voigt or Burger model. It should be noted that most of the current studies in viscoelastic materials are based on some of these mechanical models. In words of Kiasat [3], the best model to represent linear viscoelastic materials is the Boltzmann principle, also known as hereditary integral model, because the material behavior is taken in arbitrary infinitesimal time steps. Although a variety of techniques have been presented in the literature of viscoelastic beams, the finite element and finite difference based models stands out because of their effectiveness in the approximations of the solutions, relying the formulations on the well-known Euler-Bernoulli, Timoshenko, or more recently, the Reddy beam theory.
A frequently path taken by many researchers is to employ the Laplace transform in conjunction with finite elements to transform the temporal variables from a time domain t to a fictitious domain s with no convolution integral. A limitation with this procedure is that to obtain a accurate numerical solution, a very efficient inversion is required. Another prominent approach is the correspondence principle, which relates the viscoelastic problem with a similar elastic problem (same geometry and same boundary conditions). However, this approximation is only applicable with very suitable boundary conditions. An example on how this approach can be useful to study the behavior of a beam is found in the works of Martin [7, 8]. The author proposed an algorithm based on the Galerkin's method, the variational iteration method and the Laplace transform techniques to analyze the dynamic behavior of an Euler-Bernoulli beam. Then the quasi-static solution is obtained applying the correspondence principle.
On the other hand, as with elastic materials, viscoelastic structures -like beams or plates- are prone to show locking when the thickness goes to zero. A well-known solution to this issue in the elastic case is to use reduced integration in the finite element approximation. The research carried out by Arnold [4] show the bases to avoid locking on a Timoshenko beam by using a mixed formulation and reduced integration in the shear term. An example of this technique is found in Payette and Reddy [5, 6], who studied a locking-free on a non-linear Euler-Bernoulli and Timoshenko beam subjected to large displacements, moderate rotations and small strains, where the discretization is done using high-order Lagrangian interpolation. A fully dynamical system is obtained using the trapezoidal rule in the time integral.
According to the researches described above, in this work we use a constitutive law in a hereditary form to analyze the dynamic and quasi-static behavior of an isotropic linear viscoelastic

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Timoshenko beam model for which a thickness-dependent parameter is introduced. A locking-free finite element formulation using low-order elements is proposed, where the hereditary integral is approximated using a recurrence formula. All the approximations are accompanied with numerical experiments to observe the convergence of the method and the behavior of the system as the beam gets thinner.

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Reconstrucciones WENO de orden incondicionalmente alto e indicadores de suavidad eficientes

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Abstract

Este trabajo aborda la pérdida de orden alrededor de puntos críticos que sufren las reconstrucciones WENO comúnmente conocidas en la literatura, mediante un análisis teórico en profundidad. Una vez localizado el foco del problema, proponemos el rediseño de estas reconstrucciones a través de dos alternativas para conseguir el orden óptimo de forma incondicional [1], partiendo en sendos casos del diseño de pesos propuesto por Yamaleev-Carpenter [5]: la primera de ellas consiste en la inclusión de un nodo adicional en el *stencil* de reconstrucción; la segunda de ellas se basa en el cómputo de una cantidad adicional en el *stencil* de nodos original, que es utilizada posteriormente en el cálculo de los pesos no lineales.

Para reconstrucciones de tercer orden, la única alternativa que puede aplicarse es la primera, es decir, añadiendo un nodo adicional en el *stencil*. Asimismo, requiere un diseño de los pesos no lineales diferente, por lo que precisa de un análisis por separado [2]. En este caso particular, la mejora obtenida desde el punto de vista numérico es notable, obteniéndose unos resultados de mayor resolución con un coste computacional similar al de las reconstrucciones WENO de tercer orden tradicionales.

Finalmente, el nuevo diseño de pesos WENO con orden incondicionalmente óptimo permite el uso de indicadores de suavidad más sencillos y eficientes [3] que los típicamente empleados en este tipo de reconstrucciones [4]. Tanto los resultados teóricos como los resultados numéricos arrojan que estos nuevos indicadores de suavidad también generan orden incondicionalmente óptimo y que los resultados numéricos son muy similares a los obtenidos con los indicadores de suavidad tradicionales [4], con un coste computacional inferior.

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*Parcialmente financiado por Proyecto Fondecyt 3170077, e-mail: `dzorio@ci2ma.udec.cl`
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