1 Ecuaciones Diferenciales

1. **Expositor: Salomón Alarcón Araneda** [1]
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Título: Boundary blow-up problems involving the p-Laplacian in domains exhibiting a corner.

Resumen: We prove existence, uniqueness, and asymptotic behavior of solutions to the problem

$$\begin{cases} -\Delta_p u + \lambda(x) f(u) + \eta(x)|u|^{p-2}u = g(x) \quad \text{in} \quad \Omega, \\ \lim_{\mathrm{d}(x,\partial\Omega)\to 0} u(x) = \infty, \end{cases}$$

where Ω is a domain in $\mathbb{R}^N$, $N \geq 2$, which satisfies the exterior cone condition, $\mathrm{d}(\cdot, \partial\Omega)$ is the distance function to the boundary $\partial\Omega$, f is a function satisfying the Keller-Osermann condition associated with the p-Laplacian operator, $\lambda, \eta \in C(\Omega)$, $g \in L^\infty(\Omega) \cap W^{1,p'}_{\mathrm{loc}}(\Omega)$, with $g \geq 0$ in Ω, and $p \geq 2$.

Joint work with:
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Patricio Cerda[3], Departamento de Matemática y Ciencia de la Computación, Universidad de Santiago de Chile, Santiago, Chile.

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[2] Marcus, M., Veron, L., *Uniqueness and asymptotic behavior of solutions with boundary blow-up for a class of nonlinear elliptic equations*, Ann. Inst. H. Poincaré **14**, (1997). 237 - 274.

2. **Expositor: David Ignacio Urrutia Vergara**[1]
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Título: Algunos resultados de derivabilidad para el Homeomorfismo de Lin.

Resumen: Consideremos dos sistemas de ecuaciones diferenciales ordinarias. En primer lugar, el sistema no lineal no autónomo

$$\dot{y} = C(t)y + B(t)y + g(y,t) \tag{1}$$

y, en segundo lugar, el sistema lineal autónomo

$$\dot{z} = -\frac{\delta}{2} I z, \tag{2}$$

donde $\delta > 0$ e $I \in M_n(\mathbb{R})$ es la identidad. Además, $C, B : \mathbb{R} \to M_n(\mathbb{R})$ y $g : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ son continuas y verifican las siguientes propiedades:

(A1) Para todo $t \in \mathbb{R}$, se tiene que $C(t) :=$diag$\{C_1(t), C_2(t), \ldots, C_n(t)\}$ y

$$C_i(t) \leq -\delta \quad \text{para cada } t \in \mathbb{R} \text{ y todo } i \in \{1, \ldots, n\}$$

(A2) La matriz $B(t)$ satisface la siguiente estimación

$$||B(t)|| \leq \frac{\delta}{4}$$

(A3) Para cada $t \in \mathbb{R}$, se tiene que $g(0,t) = 0$ y ademas,

$$||g(y_1,t) - g(y_2,t)|| \leq \frac{\delta}{4} ||y_1 - y_2|| \quad \text{para cada } t \in \mathbb{R} \text{ y cada } y_1, y_2 \in \mathbb{R}^n.$$

Considerando esas condiciones, Faxing Lin (2007) probó en [2, Lema 1] que el sistema no lineal (1) es *topologícamente equivalente* al sistema lineal (2), es decir, existe una función $H : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ que satisface lo siguiente:

(i) $\lim\limits_{y \to y_0} H(y,t) = H(y_0,t)$ y $\lim\limits_{||y|| \to \infty} ||H(y,t)||^{-1} = 0$, uniformemente respecto a t.

(ii) Para cada $t \in \mathbb{R}$ fijo, la funcioń $H_t : \mathbb{R}^n \to \mathbb{R}^n$ definida por $H_t(y) = H(y,t)$ es un homeomorfismo.

(iii) La funcioń $G : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ definida por $G(z,t) = H_t^{-1}(z)$ también verifica (i),

(iv) $t \mapsto H(y(t),t)$ es una solucioń de (2) cuando $t \mapsto y(t)$ es una solucioń de (1) y

(v) $t \mapsto G(z(t),t)$ es una solucioń de (1) cuando $t \mapsto z(t)$ es solucioń de (2).

La funcioń de equivalencia topologíca viene dada por la funcioń $H : \mathbb{R}^n \times \mathbb{R} \to \mathbb{R}^n$ definida por:

$$H(y,\tau) = \begin{cases} Y(T(y,\tau),\tau,y) \exp\left(\dfrac{\delta}{2}(T(y,\tau) - \tau)\right) & \text{para } y \neq 0 \\ 0 & \text{para } y = 0, \end{cases} \tag{3}$$

donde $t \mapsto Y(t, \tau, y)$ es la solucioń de (1) que pasa por y en $t = \tau$ y $T(y, \tau)$ verifica

$$||Y(T(y,\tau),\tau,y)||^2 = 1 \quad \text{para cada } y \in \mathbb{R} \setminus \{0\} \text{ y } \tau \in \mathbb{R},$$

donde esta funcioń $T : \mathbb{R}^n \setminus \{0\} \times \mathbb{R} \to \mathbb{R}$ se conoce como *Crossing Time*

La equivalencia topológica define una familia de homeomorfismos $H_\tau : \mathbb{R}^n \to \mathbb{R}^n$ definida por $H_\tau(y) = H(y, \tau)$ para cada $\tau \in \mathbb{R}$ e $y \in \mathbb{R}$, los cuales se llaman *Homeomorfismos de Lin.*

Curiosamente, las propiedades de derivabilidad de los homeomorfismos de Lin no han sido estudiadas en la literatura. En ese contexto, el objetivo de este trabajo consiste en determinar las condiciones necesarias para que la función H sea de clase $\mathscr{C}^k$ en $(\mathbb{R}^n \setminus \{0\}) \times \mathbb{R}$ con $k \geq 1$ y por ende, cada homeomorfismo de Lin H_τ también sea de Clase $\mathscr{C}^k$ en $\mathbb{R}^n \setminus \{0\}$. Para ello, se estudiarán condiciones suficientes para determinar la derivabilidad de la función crossing time. Para conseguir este objetivo se requerirán herramientas tales como la diferenciabilidad de soluciones con respecto a las condiciones iniciales [3], el Teorema de la función implícita y la teoría de la estabilidad asintótica uniforme [1].

Joint work with:
Gonzalo Robledo Veloso[2], Departmento de Matemáticas, Facultad de Ciencias
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[1] William A. Coppel. *Stability and Asymptotic Behaviours of Differential Equations.* Heath Boston, 1965.

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[3] Thomas C. Sideris. *Ordinary differential equations and dynamical systems.* Atlantis Press, Paris, 2013.

3. **Expositor: Panas Kalayanamit**
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Título: On the sets of extremals for a singularly perturbed quasiconvex integral.

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Resumen: Ever since its introduction by Morrey in 1952, quasiconvexity has become a central topic in calculus of variations, largely due to its close connection to weak lower semicontinuity of the associated variational integrals on Sobolev spaces. Despite its importance, quasiconvexity is an elusive condition that is hard to verify and the behaviour of minimizers (or, more generally, extremals) of a variational integral with quasiconvex integrand is far from being fully understood.

In this talk, we shall present some results regarding the compactness and size of the sets of extremals of a regularized minimization problem for a variational integral under some strict quasiconvexity assumption. These results are inspired by a recent work of Campos Cordero and Kristensen. The relationship between quasiconvexity and other closely related topics, and a few important counterexamples to the uniqueness of minimizers are also discussed.
Joint work with:
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4. **Expositor: Carolina Ana Rey**[1]
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Título: Non-local equations and an optimal Sobolev inequality on compact manifolds.

Resumen: In this talk, we deal with the theory of fractional Sobolev spaces on a compact Riemannian manifold (M, g). Our first main result shows that the fractional Sobolev spaces $W^{s,p}(M)$ introduced by Guo, Zhang, and Zhang in [1] coincide with the classical Triebel-Lizorkin spaces (which in turn coincide with the Besov spaces).

As an application, we study a non-local elliptic equation of the form

$$\mathcal{L}_K u + h|u|^{p-2}u = f|u|^{q-2}u, \tag{4}$$

where the operator $\mathcal{L}_K u$ is an integro-differential operator a little more general than the fractional Laplacian, defined on $W^{s,p}(M)$. We use some classical methods to show an existence result in the case where the non-linearity on the right-hand side of the equation (4) is sub-critical.

Our second main result is a Sobolev inequality in the critical range with an optimal constant for the fractional Sobolev spaces $W^{s,2}(M)$. This inequality gives us a sufficient existence condition for (4) with $p = 2$ and $q = 2^* = \frac{2n}{n-2s}$ the fractional critical Sobolev exponent. Joint work with:
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[2] Rey, Carolina; Saintier, Nicolas, *Non-local equations and optimal Sobolev inequalities on compact manifolds,* (2022). Preprint arXiv:2209.12357.

5. **Expositor: Juan Carlos Pozo**[1]
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Título: Nonlocal in-time telegraph equation and telegraph processes with random time.

Resumen: In this talk we will present some properties of a non-markovian version of the telegraph process. The non-markovian character of the process comes from a time-fractional evolution equation satisfied by the probability density function. As main result, we prove that the distribution of the process coincides with the distribution of a process of the form $T(|E_t|)$ where $T(t)$ is the classical telegraph process, and E_t is an inverse stable subordinator. Our results answer some problems proposed by Orsingher and Beghin in [2]

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[2] F. Orsingher and L. Beghin, *Time-fractional telegraph equations and telegraph processes with Brownian time*, Probability Theory and Related Fields **128**, (2004), 141–160.

6. **Expositor: Pedro Hernández-Llanos**[1]
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Título: Poroelastic plate model obtained by simultaneous homogenization and dimension reduction

Resumen: In this talk, the starting point of our analysis is the Biot's equations. We consider two small parameters: the thickness h of the thin plate and the pore scale ε_h which depend on h. We will focus specifically on the case when the pore size is small relative to the thickness of the plate. The main goal here is derive a model for a poroelastic plate from the $3D$ problem as h goes to 0 using two-scale convergence, simultaneous homogenization and dimension reduction techniques. The model obtained generalizes some of the results obtained by A. Marcianiak-Czchara and A. Mikelić [1].

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7. **Expositor: Jessica Trespalacios**[1]
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Título: Global Existence of Suitable Small Solutions in the Belinksi-Zakharov spacetimes, and Applications.

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Resumen: We consider the Vacuum Einstein Field equations under the Belinski-Zakharov symmetry. Depending on the chosen signature of the metric, these spacetimes contain most of the well-known special solutions in General Relativity, including black holes and cosmological-type solutions. In this paper, we prove local existence of large and global existence of small Belinski-Zakharov solutions under a non degeneracy condition. We also construct virial functionals which provide a clear description of decay of smooth global cosmological-type solutions inside the light cone. Finally, some applications are presented in the case of a kasner-type metric. [1, 2, 3, 4].

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8. **Expositor: Duvan Henao Manrique**[1]
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Título: Dipolos armónicos en elasticidad.

Resumen: Whenever the stored energy density of a hyperelastic material has slow growth at infinity (below $|F|^p$ with p less than the space dimension), it may undergo cavitation (the nucleation and sudden growth of internal voids) under large hydrostatic tension [Ball, 1982; James & Spector, 1992]. This constitutes a failure of quasiconvexity and, hence, a challenge for the existence theory in elastostatics [Ball & Murat, 1984]. The obstacle has been overcome under certain coercivity hypotheses [Müller & Spector, 1995; Sivaloganathan & Spector, 2000] which, however, fail to be satisfied by the paradigmatic example in elasticity: that of 3D neo-Hookean materials. A joint work with Marco Barchiesi, Carlos Mora-Corral, and Rémy Rodiac will be presented, where this borderline case was solved for hollow

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axisymmetric domains. Partial results leading to a solution when the axis of rotation is contained (where the dipoles found by [Conti & De Lellis, 2003] must be proved to be non energy-minimizing) will also be discussed.

Trabajo realizado junto a:
M. Barchiesi[2], **C. Mora-Corral**[3], **R. Rodiac**[4]

References

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9. **Expositor: Mario Choquehuanca**[1]
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Título: The second–order dynamic equations involving time scales.

Resumen: The study of dynamic equations on time scales is a fairly new subject , and researchs in this area is rapidly growing. It has been created in order to unify continuous and discrete analysis, and it allows a simultaneous treatment of differential and difference equations, extending those theories to so-called dynamic equations, (for a introduction consult [1, 2])

In this oportunity we present some investigations on existence and uniqueness of solutions of linear and semilinear second-order equations involving time scales. To obtain such results, we make use of exponential dichotomy and fixed point results. Also, we present some examples and applications to illustrate our main results.

Joint work with:
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10. **Expositor: Silvia Rueda Sanchez**[1]
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Título: Propiedades analíticas de operadores discretos no locales: Convexidad y Jerk.

Resumen: En esta charla hablaremos sobre resultados de convexidad para operadores discretos no locales. Estos resultados amplían los estudios actuales sobre propiedades de positividad, monotonicidad, convexidad, casos límite. Además, mostraremos nuevos conocimientos sobre tales propiedades por medio de ejemplos originales que evidencian la nitidez de los resultados. Enunciamos una respuesta plausible a la inversa de la conjetura de monotonicidad de Dahal-Goodrich. Presentaremos una nueva noción geométrica para una función de valor real de?nida en un dominiodiscreto que depende de un parámetro $\alpha \geq 2$. Daremos ejemplos para ilustrar las conexiones entre la convexidad y este nuevo concepto. Luego demostraremos dos criterios basados en el signo del operador fraccionario discreto de una función u, $\Delta^{\alpha} u$ con $2 \leq \alpha \leq 4$. Dos ejemplos muestran que los criterios dados son óptimos con respecto a la noción geométrica establecida. Nuestro método se basa en el principio de transferencia.

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11. **Expositor: Jorge González-Camus**[1]
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Facultad de Ciencias Naturales, Matemáticas y del Medio Ambiente,
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Título: Time-step heat problem on the mesh: asymptotic behavior and decay rates

Resumen: In this talk, we present results about the asymptotic behaviour and decay of the solution of the fully discrete heat problem

$$\begin{cases} \dfrac{u(nh,m)-u((n-1)h,m)}{h} = \Delta_d u(nh,m) + g(nh,m), \quad n \in \mathbb{N}, m \in \mathbb{Z}, \\ u(0,m) = f(m), \end{cases} \tag{5}$$

We show that basic properties of its solutions, such as the principle of conservation of mass, and moments are similar to the continuous case. Using the Poisson transform, we obtain estimates for the fundamental solution and we use those to study its asymptotic behaviour.

Joint work with:
Luciano Abadías[2], Departamento de Matemáticas, Instituto Universitario de Matemáticas y Aplicaciones, Universidad de Zaragoza, 50009 Zaragoza, Spain.
Silvia Rueda[3], Departamento de Matemáticas, Facultad de Ciencias, Universidad del Bío Bío, Concepción, Chile

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12. **Expositor: Ricardo Torres Naranjo**[1]
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Título: Aproximaciones de funciones por medio de ecuaciones diferenciales con argumento constante a trozos.

Resumen: En los ’70s, el matemático ucraniano *A.D.Myshkis* propuso un nuevo tipo de ecuaciones diferenciales $x'(t) = f(t, x(t), x(\rho(t)))$; donde $\rho(t)$ corresponde a un caso particular de funciones localmente constantes, por ejemplo, $\rho(t) = [t]$. Estas ecuaciones fueron llamadas **Ecuaciones diferenciales con argumento constante a trozos** (en inglés *DEPCA*). En los 2000, el también matemático ucraniano *M.U. Akhmet* generalizó el trabajo de Myshkis definiendo los sistemas

$$x'(t) = f(t, x(t), x(\gamma(t))), \tag{6}$$

donde $\gamma(t)$ es un **argumento constante a trozos del tipo generalizado**. Estas ecuaciones fueron llamadas *Ecuaciones diferenciales con argumento constante a trozos del tipo general* ó Differential Equations with Piecewise Constant Argument of Generalized Type (*DEPCAG*).

Tal función γ divide todo intervalo I_n en dos partes: una avanzada y otra retardada con respecto a la función $f(t) = t$, i.e $I_n = I_n^+ \bigcup I_n^-$, donde $I_n^+ = [t_n, \zeta_n]$ y $I_n^- = [\zeta_n, t_{n+1})$.

Las ecuaciones DEPCAG son bastante especiales, dado que ellas poseen soluciones continuas aún cuando $\gamma(t)$ no lo es. En ambos extremos de cada intervalo de constancia de γ, una ley recursiva es producida y ella define una ecuacion en diferencias finitas. Debido a esto, las ecuaciones DEPCAG son también llamadas del tipo híbridas, ya que ellas combinan dinámica discreta y continua.

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Consideremos ahora $\gamma(t) = \left[\frac{t}{h}\right] h + \beta h$, donde $\beta \in [0,1]$ y $h > 0$.

Este argumento tiene valor constante $\gamma(t) = (n+\beta)h$ en todo intervalo de la forma $I_{n(t)} = [nh, (n+1)h)$, por lo que se tiene $I_n^+ = [nh, (n+\beta)h)$ y $I_n^- = [(n+\beta)h, (n+1)h)$.

Sean los sistemas

$$\begin{aligned} x'(t) &= \Lambda(t)x(t), \\ x_h'(t) &= \Lambda(t)x_h\left(\left[\frac{t}{h}\right] h + \beta h\right) \end{aligned}$$

donde $\Lambda(t)$ es una matriz $n \times n$ con entradas uniformememte continuas y localmente integrables. Entonces, bajo ciertas condiciones de estabilidad, se tiene:

$$x_h(t) \xrightarrow[h\to 0]{} x(t)$$

En el caso escalar con coeficientes constantes,

$$z'(t) = az\left(\left[\frac{t}{h}\right] h + h\beta\right)$$

donde $\beta h|a|,\ (1-\beta)h|a| \neq 1$, se tiene que su solución satisface

$$\left(\frac{1+(1-\beta)ha}{1-\beta ha}\right)^{\left[\frac{t}{h}\right]-\left[\frac{t_0}{h}\right]} \xrightarrow[h\to 0]{} e^{a(t-t_0)}$$

Con lo anterior, se recuperan algunos esquemas clásicos de aproximación de soluciones, tales como: ($\beta = 0$ Esquema clásico de Euler), ($\beta = 1$ Esquema clásico avanzado de Euler y ($\beta = \frac{1}{2}$ esquema clásico del trapezoide).

Mostraremos algunas situaciones interesantes de aproximación de funciones elementales de manera sencilla ilustrando el método necesario para cada caso. También mostraremos cómo generar aproximantes de soluciones de ecuaciones diferenciales lineales de orden n.

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13. **Expositor: Harold Bustos**[1]
Facultad de Ingeniería
Universidad Austral de Chile
Valdivia, Chile.

Título: Classes of pseudo-differential operators of vector valued symbols

Resumen: During last decades M. Ruzhansky and V. Turunen extended ideas of global pseudo-differential operators on $\mathbb{R}^n$ to certain Lie groups [3, 4, 5]. These ideas can be used to generalize pseudo-differential operators and its symbol classes to more general spaces. In this talk we will discuss a way to extend operators and symbols using Fourier multipliers. We will also mention some applications to elliptic equations and its relation with quantization.

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14. **Expositor: Hanne Van Den Bosch**
Departamento de Ingeniería Matemática & CMM
Universidad de Chile
Santiago, Chile

Título: Mixing and oscillations in the linearized Vlasov-Poisson system

Resumen: The Vlasov–Poisson system is an effective model for a large number of particles interacting through a self-consistent Coulomb potential. When this potential is attractive, it is a common model for galaxies or clouds of dusts. Steady state solutions can be constructed by a variational approach, and in many particular cases, they are known to be stable against small perturbations. The question of assymptotic stability, i.e., do perturbations of a steady state relax to *some* steady state or oscillate indefinetely, is open in this context. This motivates the study of the linearized operator around a steady state. The behaviour of this linearized equation is closely related to the behaviour the phase space - mixing properties of the non-interacting model. I will review the state of the art and discuss some recent work on this topic.

Joint work with: Matías Moreno and Paola Rioseco

15. **Expositor: Andrei Rodríguez Paredes**
Departamento de Matemática
Universidad de Concepción
Concepción, Chile [1]
Título: Large-time behavior of viscous Hamilton-Jacobi equations.

Resumen: We determine the large-time behavior of unbounded solutions for the so-called *viscous Hamilton Jacobi equation*, $u_t - \Delta u + |Du|^m = f(x)$, in the *quadratic* and *subquadratic* cases (i.e., for $1 < m \leq 2$), with a particular focus on allowing the greatest generality possible for the source term f and the prescribed initial data regarding growth at infinity. The lack of a comparison principle for the associated *ergodic problem* is overcome by proving that a *generalized simplicity* holds for sub- and supersolutions of the ergodic problem. Moreover, as the uniqueness of solutions of the parabolic problem remains open in the current setting, our result on large-time behavior holds for *any* solution, even if multiple solutions exist.

Joint work with:
Alexander Quaas[2], Departamento de Matemática, Universidad Técnica Federico Santa María, Valparaíso, Chile.

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[2] Partially supported by Fondecyt Grant 1190282 and Programa Basal, CMM, U. de Chile., e-mail: alexander.quaas@usm.cl

16. **Expositor: Gerard Olivar-Tost**[1]
Department of Natural Sciences & Technology
University of Aysen
Coyhaique, Chile.

Título: A piecewise smooth mathematical model for the dynamics of an energy market

Resumen: In this communication, we propose a mathematical model to describe the dynamics of an electric power market (without stock) through a low-dimensional system of ordinary differential equations, which considers significant variables such as supply, demand, the reserve margin, and the investment, which is defined through a piecewise-smooth function of the expected return.

The resulting model (a Filippov system) is qualitatively analyzed using stability concepts, considering the trajectories that approach the switching surfaces. The different conditions give rise to non-standard bifurcations, described in the literature [1, 2] and give rise to different scenarios, which are reinterpreted again in the energy market framework

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