8 Geometría

1. **Expositor:** Saúl Quispe.
   Título: Ends space of the fiber product over infinite-genus Riemann surfaces.
   Afiliación: Universidad de La Frontera.
   Resumen: Considering non-constant holomorphic maps $\beta_i : S_i \to S_0$, $i \in \{1, 2\}$, between non-compact Riemann surfaces for which it is associated its fiber product $S_1 \times_{(\beta_1,\beta_2)} S_2$. With this setting, in this talk we relate the ends space of the fiber product $S_1 \times_{(\beta_1,\beta_2)} S_2$ to the ends space of its normal fiber product. Moreover, we provide conditions on the maps β_1 and β_2 to guarantee connectednes on the fiber product. From these conditions, we link the ends space of fiber product with the topological type of the Riemann surfaces S_1 and S_2. Finally, we study the fiber product over infinite hyperelliptic curves and discuss its connectedness and ends space [1].

   Joint work with:
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[2] Hidalgo, R. A; Reyes-Carocca, S.; Vega, A., *Fiber product of Riemann surfaces, Automorphisms of Riemann surfaces, subgroups of mapping class groups and related topics*, Contemp. Math. Amer. Math. Soc., 2022.

2. **Expositor:** Yerika Marín Montilla.
   Título: The symmetric hyperbolic genus of quasi-abelian group.
   Afiliación: Universidad de La Frontera.
   Resumen: Let $QA_n = \langle x, y : x^{2^{n-1}} = y^2 = 1, [x, y] = x^{2^{n-2}} \rangle$ be the **quasi-abelian group** of order 2^n, where $n \geq 4$. It is well known that QA_n acts as a group of automorphisms of some closed Riemann surface. The **symmetric hyperbolic genus** of QA_n is the minimal genus $\sigma^{hyp}(QA_n) \geq 2$ of suitable Riemann surfaces over which QA_n acts as a group of conformal/anticonformal automorphisms, and admitting anticonformal ones [1].

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In this talk we will describe the actions of the group QA_n on closed Riemann surfaces, and as a consequence, we get that the symmetric hyperbolic genus $\sigma^{hyp}(QA_n)$ is 2^{n-3}. Finally, we study the uniqueness action problem on this minimal genera [2].

This is part of my Ph.D. Thesis, under the advisers:

Rubén A. Hidalgo[3] and **Saúl Quispe**[4].
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[2] R. A. Hidalgo, Y. Marín Montilla and S. Quispe. Quasi-abelian group as automorphism group of Riemann surfaces, Preprint 2023.

3. **Expositor:** Sebastián Reyes-Carocca.
Afiliación: Universidad de Chile.
Título: Sobre ciertas subvariedades del espacio de módulos de superficies de Riemann.
Resumen: En esta charla consideraremos ciertas subvariedades del espacio de módulos de superficies de Riemann compactas determinadas por la especificación de acciones de grupos. Discutiremos el problema general de determinar cuáles de entre ellas son subvariedades normales, daremos ejemplos y presentaremos algunos resultados recientes.

   Este es un trabajo en progreso en conjunto con Jennifer Paulhus (Grinnell College) y Anita M. Rojas (Universidad de Chile).

4. **Expositor:** Antonio Laface.
Afiliación: Universidad de Concepción.
Título: Ample bodies and Terracini loci of projective varieties.
Resumen: Let $X \subseteq \mathbb{P}^N$ be an irreducible projective variety. The h-Terracini locus of X, denoted by $T_h(X)$, parametrizes unordered h-uples of distinct points $x_1, \ldots, x_h \in X$ at which the tangent spaces span a linear space of dimension smaller than expected. Terracini loci have been

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introduced in [1] and the studied for several relevant varieties such as Veronese and Segre varieties.

We associate to a projective variety X a geometric object $\mathscr{A}_X$ that we call the ample body of X. This is the convex hull of ample divisor classes of X. When the Mori cone of X is rational polyhedral $\mathscr{A}_X$ turns out to be a polyhedron which is the Minkowski sum of a rational polytope A_X and of the nef cone of X. In many case we manage to control the geometry of $\mathscr{A}_X$ and to use it to prove emptiness results for Terracini loci. Our main result states that if $X_P \subseteq \mathbb{P}^N$ is a smooth projective toric variety defined by a lattice polytope $P \subseteq \mathbb{Q}^n$, whose ample body is a normal lattice polytope, then $T_h(X)$ is empty if and only if all edges of P have lattice length at least $2h-1$, see [2].

Joint work with:
Alex Massarenti[5], Università di Ferrara.

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5. **Expositor:** Paola Comparin.
Título: Acciones de grupos en superficies K3 y variedades IHS.
Afiliación: Universidad de La Frontera.
Resumen: La clasificación de grupos finitos maximales que actúan de forma simpléctica en una superficie K3 se debe a Mukai [2], quien mostró que el grupo de orden más grande de este tipo es el grupo de Mathieu M_{20}, cuyo orden es 960. C. Bonnafé y A. Sarti en [1] muestran las posibles extensiones de M_{20} que actuan en una superficie K3 y prueban que existen 3 superficies K3 con acción de estas extensiones. En la charla hablaremos de superficies K3 que admiten la acción de M_{20} y mostraremos como el mismo problema se estudia también en el caso de variedades simplécticas holomorfas. Se trata de trabajos en común con Pablo Quezada (UFRO) y Romain Demelle (Université de Poitiers).

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6. **Expositor:** Angel Carocca.
Afiliación: Universidad de La Frontera.
Título: Some properties of the Galois closure of covers.
Resumen: In this note we will present some results related with the Galois group of covers $\varphi \circ \psi$ of curves $\mathcal{Y} \xrightarrow{\psi} \mathcal{X} \xrightarrow{\varphi} \mathbb{P}^1$, where φ is a cover of degree $\mathfrak{m}$ and $\psi : \mathcal{Y} \to \mathcal{X}$ is an abelian unramified Galois cover of degree $\mathfrak{r}$.

We will show that the Galois group $\mathcal{G}$ of the Galois closure $\mathcal{Z}$ of $\varphi \circ \psi$ is of the form
$$\mathcal{G} \cong \mathcal{K} \rtimes \mathcal{U}$$
where $\mathcal{K} \leq (\mathcal{H}/\mathcal{N})^{\mathfrak{m}}$, with $\mathcal{N}$ and $\mathcal{H}$ the corresponding subgroups of $\mathcal{G}$ such that $\mathcal{Z}_{\mathcal{N}} \cong \mathcal{Y}$ and $\mathcal{Z}_{\mathcal{H}} \cong \mathcal{X}$.

7. **Expositor:** Rubí Rodríguez.
Título: Some properties of the Galois closure of covers II.
Afiliación: Universidad de La Frontera.
Resumen: In this note we will present a geometric construction of the Galois closure and the Galois group of covers $\varphi \circ \psi$ of curves $\mathcal{Y} \xrightarrow{\psi} \mathcal{X} \xrightarrow{\varphi} \mathbb{P}^1$, where φ is a cover of degree $\mathfrak{m}$ and $\psi : \mathcal{Y} \to \mathcal{X}$ is an abelian unramified Galois cover of degree $\mathfrak{r}$.

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8. **Expositor:** Pedro Montero.
Título: Automorfismos de estructuras de Hodge de hipersuperficies de Klein. **Afiliación:** Universidad Técnica Federico Santa María.
Resumen: La estructura de Hodge de una variedad algebraica proyectiva y suave es un importante invariante que, en muchos casos, se espera

que determine la variedad en sí misma. Esta expectativa se conoce como el "principio de Torelli" y se basa en un teorema clásico de Torelli para curvas algebraicas y sus Jacobianas. En esta charla, nos centraremos en las hipersuperficies de Klein en el espacio proyectivo (que son una generalización a dimensiones superiores de la famosa curva cuártica de Klein) y explicaremos cómo determinar su grupo de automorfismos y, en algunos casos, cómo calcular el grupo de automorfismos de la estructura de Hodge polarizada asociada. Estos resultados proporcionan nueva evidencia positivas para el principio de Torelli para cúbicas de dimensión 5 y cuárticas de dimensión 3, para las cuales podemos asociar una variedad abeliana principal polarizada llamada Jacobiana Intermedia.

Trabajo conjunto con:
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9. **Expositor:** Jorge Duque.
Afiliación: Universidad de Chile.
Título: About a Movasati's conjecture: A counterexample to the Hodge conjecture?
Resumen: The Hodge conjecture is one of the Clay Institute's seven millennium problems. Whoever solves it will win a million dollars. If instead of trying to solve this difficult conjecture you try to solve a Movasati's conjecture and win 10000 dollars? This conjecture is the first GADEPS problem [1].

In [2] it was proved that the locus of general hypersurfaces containing two linear cycles whose intersection is of dimension $m < \frac{n}{2} - \frac{d}{d-2}$, corresponds to the Hodge locus of any integral combination of such linear cycles. The not covered cases are finite and the Movasati's conjecture is about one of these cases. This conjecture predicts that for $d = 3$ and $m = \frac{n}{2} - 3$ the dimension of the Hodge locus of any integral combination of two linear cycles whose intersection is m, is equal to the dimension of the locus of general hypersurfaces containing such linear cycles plus one and further that this Hodge locus is smooth. This would be a strange phenomenon given what is already known from [2]. We compute the second order invariant of the IVHS associated to this Hodge locus on the Fermat point. This gives some evidence of the non-triviality of Movasati conjecture. On the other hand, techniques to calculate higher-order approximations of the Hodge locus need to be developed. The algebraic cycle in this conjecture is an example of Join algebraic cycle. We can understand the polynomial associated to the Join cycle as well as the structure of its associated Artinian Gorenstein ideal, but it is not obvious that in this context its Hodge locus is smooth.

In this talk we will make a small trip around this conjecture.

Joint work with: Roberto Villaflor and Hossein Movasati.

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10. **Expositor:** Anita Rojas.
Afiliación: Universidad de Chile.
Título: On completely decomposable Jacobian varieties.
Resumen: Completely decomposable Jacobian varieties (cdjv) are Jacobian varieties which are isogenous to a product of elliptic curves. These are objects that have been in the interest of geometers and algebrists since long time ago, since completely decomposable abelian varieties have interesting properties [1]. In 1993, Ekedahl and Serre [2] found cdjv in several dimensions. Nevertheless they left several gaps, where it is unknown whether there are such kind of Jacobians. To fill these gaps has been an interesting question since then. Moreover, they asked in that paper whether there are cdjv in every dimension and if they stop appearing after some high dimension. If this is the case, what is that bound. These are still open questions nowadays. In 2011, Moonen and Oort put several questions regarding special subvarieties in the moduli space $\mathcal{A}_g$ of principally polarized abelian varieties. Among them we find [3, Question 6.7], *for which $g \geq 2$ there exists a positive dimensional subvariety Z in the closure of the Jacobian locus $\mathcal{T}_g^0$, and intersecting it, such that the generic point of Z is completely decomposable.* In this talk, we review some of the known results on this matter, we present some tools, both theoretical and computational, to go beyond what is done up to now, and we apply them to show how we have found a cdjv on dimension 101, filling in this way an

Ekedahl-Serre's gap left open in [4].

Joint work with:
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11. **Expositor:** Gary Martinez.
Afiliación: Universidad de Chile.
Título: Decomposition of Jacobians of Generalized Fermat curves.
Resumen:

Let F be an algebraically closed field of characteristic $q \geq 0$. For $n, k \in \mathbb{N}$, with $n \geq 2$ and $(k, q) = 1$, a generalized Fermat curve of type (n, k), denoted by $X_{(n,k)}$, is a non-singular irreducible projective algebraic curve defined over F such that $\mathrm{Aut}(X_{(n,k)})$ has a subgroup E isomorphic to $(\mathbb{Z}/k\mathbb{Z})^n$ and $X_{(n,k)}/E$ is isomorphic to the projective line with exactly $(n+1)$ cone points, each one of order k. This group E is called a generalized Fermat group of type (n, k).

A particular case of generalized Fermat curves are those of type $(n, 2)$, which correspond to Humbert-Edge curves. In [1], the authors give a decomposition of the jacobian varieties of Humbert-Edge curves and, furthermore, they prove that each factor on such decomposition is a Prym-Turin variety for the curve. In this work we generalized the decomposition given in [1] to the case of generalized Fermat curves of type (n, k) with k a prime number and prove that none of the factors on the decomposition is a Prym-Tyurin variety for the respective curve if $k \geq 5$.

[8]Partially supported by ANID Fondecyt 1230708, e-mail: rubi.rodriguez@ufrontera.cl

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12. **Expositor:** Maximiliano Leyton.
Título: Resoluciones simultaneas de deformaciones $\mu^\star$-constantes.
Afiliación: Universidad de Talca.
Resumen: Una deformación de una hipersuperficie V sobre un germen de variedad lisa (S, o) es un morfismo plano $\rho : W \to (S, o)$ tal que la fibra sobre o es V. Es natural preguntarse si una deformación de V admite una resoluciones simultanea. Es decir, un morfismo, $\pi : \widetilde{W} \to W$, birracional y propio relativo a S tal que cada fibra sobre S es una resolución de singularidades. Existen varias nociones de resoluciones simultaneas, por nombrar algunas: resolución simultanea muy débil, débil, fuerte, incrustada (ver [Tei80], [NÍ13]).

La existencia de una resolución simultanea está fuertemente relacionada con invariantes topológicos de la singularidad de V y por ende con la topología de V. Por ejemplo: En el caso que W es una deformación de una singularidad aislada de hipersuperficie de dimensión 2, H. Laufer prueba que W admite una resolución simultanea fuerte si y sólo W es una deformación $\mu^\star$-constante, donde $\mu^\star$ es la secuencia de Milnor-Tessier (ver [Lau87]).

En el caso que W es una deformación no degenerada de una singularidad aislada de hipersuperficie no degenerada de dimensión arbitrara, la existencia de una resolución simultanea incrustada de W es equivalente a que la deformación sea del tipo μ-constante, donde μ es el número de Milnor (ver [LAMS22]). Un hecho importante de este tipo de resoluciones, que las diferencia de otros tipos de resoluciones simultaneas, es la existencia de un espacio ambiente donde la transformada total de la deformación tiene intersecciones transversales.

Todavía es un problema abierto caracterizar, en dimensión arbitraria, las resoluciones simultaneas incrustadas que corresponden a deformaciones $\mu^\star$-constantes no degeneradas. En esta charla hablaremos de recientes avances obtenidos al respecto.

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